1) Use the power series for $\frac{1}{1-x}$ to find a power series for $\ln|1-x|$. Write out three simplified terms and also write with summation notation.

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots = \sum_{k=0}^{\infty} x^k$$

$$\int \frac{1}{1-x} \, dx = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots = \sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1},$$

$$-\ln|1-x|$$

$$\sum_{k=0}^{\infty} \frac{\ln(1-x)}{1-x} = -x - \frac{x^2}{2} - \frac{x^3}{3} + \cdots$$

$$= \sum_{k=0}^{\infty} (-1)^{k+1} \frac{x^{k+1}}{k+1}$$

2) Use the power series in 1) to find a power series for $\int x \ln|1-x| \, dx$. Write out three simplified terms and also write with summation notation.

$$x \ln|1-x| = x (-x - \frac{x^2}{2} - \frac{x^3}{3} + \cdots)$$

$$= -x^2 - \frac{x^3}{2} - \frac{x^4}{3} + \cdots$$

$$= \sum_{k=0}^{\infty} (-1)^{k+2} \frac{x^{k+2}}{k+1}$$

$$\int x \ln|1-x| \, dx = -\frac{x^3}{3} - \frac{x^4}{2.4} - \frac{x^5}{3.5} + \cdots$$

$$= \sum_{k=0}^{\infty} (-1)^k \frac{x^{k+3}}{(k+1)(k+3)}$$
1) Use the power series for \( \frac{1}{1-x} \) to find a power series for \( \frac{1}{(1-x)^2} \). Write out three simplified terms and also write with summation notation.

\[
\frac{1}{1-x} = 1 + x + x^2 + \cdots = \sum_{k=0}^{\infty} x^k
\]

\[
\frac{1}{(1-x)^2} = \frac{d}{dx} \left( \frac{1}{1-x} \right) = 0 + 1 + 2x + \cdots = \sum_{k=0}^{\infty} k x^{k-1}
\]

2) Use the power series in 1) to find a power series for \( \int \frac{x}{(1-x)^2} \, dx \). Write out three simplified terms and also write with summation notation.

\[
\frac{x}{(1-x)^2} = x + 2x^2 + \cdots = \sum_{k=0}^{\infty} k x^k
\]

\[
\int \frac{x}{(1-x)^2} \, dx = \frac{x^2}{2} + \frac{2x^3}{3} + \cdots = \sum_{k=0}^{\infty} \frac{k}{k+1} x^{k+1}
\]
1) Use the power series for \( \frac{1}{1-x} \) to find a power series for \( \frac{1}{1+x^2} \). Write out three simplified terms and also write with summation notation.

\[
\frac{1}{1-x} = 1 + x + x^2 + \ldots = \sum_{k=0}^{\infty} x^k
\]

\[
\frac{1}{1+x^2} = 1 + (-x^2) + (-x^4) + \ldots = 1 - x^2 + x^4 - \ldots = \sum_{k=0}^{\infty} (-1)^k x^{2k}
\]

2) Use the power series in 1) to find a power series for \( \int \frac{1}{1+x^2} \, dx \). Write out three simplified terms and also write with summation notation.

\[
\int \frac{1}{1+x^2} \, dx = \int 1 - x^2 + x^4 - \ldots \, dx
\]

\[
= x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots
\]

\[
= \sum_{k=0}^{\infty} (-1)^k \int x^{2k} \, dx
\]

\[
= \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}
\]