1) (10 points) Suppose $S$ is a sample space and $E$ and $F$ are events such that $P(E) = .5$, $P(F) = .2$, and $P(E \cap F) = .1$.

a) (4 points) Find $P(E \cup F)$.

b) (4 points) Find $P(E \cap F^c)$.

c) (2 points) Are $E$ and $F$ mutually exclusive? Explain your answer using complete sentences.
2) (20 points) Consider 2 classes, the first with 12 students, and the second with 10 students. From this set of students, a group of 4 students (that is, an unordered set of 4 students) is to be selected.

a) (5 points) How many distinct groups are possible?

b) (5 points) How many distinct groups are possible if all the students come from the first class?

c) (5 points) How many distinct groups are possible if the group includes two students from each class?

d) (5 points) How many choices are possible if at least one student must come from the first class?
3) (20 points) Two standard 6-sided dice, one red and one green, are rolled. Let $R$ be the value the red die lands on, and $G$ be the value the green die lands on.

a) (10 points) What is the probability that $R + G = 10$?

b) (10 points) What is the probability that $R - G = 2$?
4) (15 points) A standard deck of cards has 13 denominations (ace, two, three, ..., queen, king) and 4 suits (diamonds, clubs, spades, hearts) for a total of 52 cards. In the game of *Russell*, each player is dealt 6 cards.

a) (5 points) What is the probability that you are dealt a flush (this occurs when all 6 cards are the same suit)?

b) (5 points) What is the probability that you are dealt a “full-mansion” (two cards of one denomination and 4 cards of the other denomination)?

c) (5 points) What is the probability that you are dealt a “three-pair” (this occurs when the cards have denominations a, a, b, b, c, c, and a, b, c are all distinct)?
5) (20 points) Pictured below is a $4 \times 6$ grid. Starting at point $A$, you can only move up or to the right. This continues until you reach point $B$.

a) (12 points) Using the described moves, how many paths are there from point $A$ to point $B$?

b) (8 points) Using the specified moves, how many paths are there from point $A$ to point $B$ which do not go through point $C$?
6) (10 points) You are choosing a committee of 8 people from a group of 20. There are two people who are fighting and cannot serve on the committee together. How many possible committees are there?

a) (5 points) Student A answers \( \binom{18}{7} + \binom{18}{7} + \binom{18}{8} \). This answer is correct. Explain how this is the right answer by describing what each term of the answer represents. *Be sure to use complete sentences.*

b) (5 points) Student B answers \( \binom{20}{8} - \binom{18}{6} \). This answer is also correct. Explain how this is the right answer by describing what each term of the answer represents. *Be sure to use complete sentences.*
7) (10 points) The following equality is true for all integers \( n > 0 \) and integers \( 0 \leq r \leq n \):

\[
\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}
\]

a) (5 points) Show the equality is true by using an algebra argument. \textit{Hint: Find a common denominator on the right hand side of the equation.}

b) (5 points) Show the equality is true by using a probability argument (a probability argument is something like your answer to #6).