Homework Quiz 3

Tuesday, July 17th

Name: SOLUTIONS

Answer all three questions. You may use your HW on this quiz. You may not use a calculator.

1) (2 points) [Ch 3, #5] An urn contains 6 white balls and 9 black balls. If 4 balls are to be randomly selected without replacement, what is the probability that the first 2 selected are white and the second two black?

\[
\frac{6}{15} \cdot \frac{5}{14} \cdot \frac{9}{13} \cdot \frac{8}{12}
\]

2) (4 points) [Ch 3, #6] Consider an urn containing 12 balls, of which 8 are white. A sample size of 4 is to be drawn without replacement. What is the conditional probability that the first and third balls drawn will be white given that the sample size contains exactly 3 white balls?

Hard way: E is 1st + 3rd balls are white
F is sample size contains exactly 3 white balls.

Then

\[
P(E|F) = \frac{P(\text{EF})}{P(F)} = \frac{\frac{8 \cdot 7 \cdot 6 \cdot 4 \cdot 2}{12 \cdot 11 \cdot 10 \cdot 9}}{\binom{8}{3}} \cdot \frac{9}{14} = \ldots = \frac{1}{2}
\]

OR

Easy way. Since 3 white are chosen, there are 4 possibilities:

- w w w - 2 of them satisfy E;
- w w w -
- w w w -
- w w w -

so \(\frac{3}{4} = \frac{1}{2}\)
3) (4 points) [Ch 3, #9] Consider 3 urns:
- Urn A contains 2 white balls and 4 red balls.
- Urn B contains 8 white balls and 4 red balls.
- Urn C contains 1 white ball and 3 red balls.

What is the probability that the ball chosen from urn A was white given that exactly 2 balls were selected?

\[
P(A = w \mid 2 \text{ whites}) = \frac{P(A = w, B = w, C = w) + P(A = w, B = w, C \neq w)}{P(2 \text{ white})}
\]

\[
= \frac{\frac{2}{6} \cdot \frac{2}{3} \cdot \frac{3}{4} + \frac{2}{6} \cdot \frac{4}{12} \cdot \frac{1}{4}}{\frac{1}{3} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{4}}
\]

\[
= \frac{\frac{1}{3} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{4}}{\frac{1}{3} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{4}}
\]