Homework Quiz 4

Thursday, July 19th

Name: ____________ Solutions

Answer all three questions. You may use your HW on this quiz. You may not use a calculator.

1) (4 points) [Ch 4, #13] A salesman has scheduled two appointments to sell encyclopedias. His first appointment will lead to a sale with probability .3 and his second will lead to a sale with probability .6. Any sale made is equally likely to be either for the deluxe model, which costs $1000, and the standard model, which costs $500. Let \( p(a) \) be the probability mass function. Find:

a) (1 point) \( p(0) \)

\[
p(0) = P(X=0) = (0.7)(0.4) = 0.28
\]

b) (2 points) \( p(1000) \)

\[
p(1000) = P(2 \text{ standard} + 1 \text{ deluxe}) = (0.3) \cdot \frac{1}{2} \cdot (0.6) \cdot \frac{1}{2} + \left[ (0.3) \cdot \frac{1}{2} \cdot (0.4) + (0.7)(0.6) \cdot \frac{1}{2} \right]
\]

\[
= \frac{0.18}{4} + \frac{0.12}{2} + \frac{0.42}{2} = 0.045 + 0.06 + 0.21 = 0.315
\]

c) (1 point) \( p(1250) \)

\[
0
\]
2) (4 points) [Ch 3, #23] Urn I contains 2 white balls and 4 red balls, whereas Urn II contains 1 white ball and 1 red ball. A ball is randomly chosen from Urn I and put into Urn II, and a ball is then randomly chosen from Urn II.

a) (3 points) What is the probability that the ball selected from Urn II is white?

\[
P(w) = P(w \mid w \text{ transf}) P(w \text{ transf}) + P(w \mid R \text{ transf}) P(R \text{ transf})
\]
\[
= \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3}
\]
\[
= \frac{4}{9}
\]

b) (3 points) What is the conditional probability that the transferred ball is white given that the white ball is selected from Urn II?

\[
P(w \text{ transf} \mid w) = \frac{P(w \text{ transf} \cap w)}{P(w)} = \frac{P(w \text{ transf}) \cdot P(w \text{ transf})}{P(w)}
\]
\[
= \frac{\frac{2}{3} \cdot \frac{1}{3}}{\frac{4}{9}} = \frac{1}{2}
\]

3) (3 points) [Ch 4, #4] Five men and five women are ranked according to their scores on an examination. Assume that no two scores are alike and all 10! possible rankings are equally likely. Let \( X \) denote the highest ranking achieved by a woman. (For instance, \( X = 1 \) if the top ranked person is female.) Find:

a) \( P\{X = 1\} \)

\[
\frac{\binom{4}{4}}{\binom{10}{5}} = \frac{9!}{4!5!} \cdot \frac{10!}{5!5!} = \frac{1}{2}
\]

b) \( P\{X = 6\} \)

\[
\frac{\binom{4}{4}}{\binom{10}{5}} = \frac{1}{\binom{10}{5}} = \frac{5\cdot4\cdot3\cdot2}{10\cdot9\cdot8\cdot7\cdot6}
\]

c) \( P\{X = 9\} \)

\[\square\]