Homework Quiz 8

Ms362K

Name: Solutions

Answer the question. You may use your HW on this quiz. There is no need to simplify your answers.

1. (4 points) (p. 271, # 9bc) The joint probability density function of \( X \) and \( Y \) is given by:

\[
f(x, y) = \frac{6}{7} \left( x^2 + \frac{xy}{2} \right) \quad 0 < x < 1 \quad 0 < y < 2.
\]

b) Compute the density function of \( X \).

\[
f_X(x) = \int_{-\infty}^{\infty} f(x, y) \, dy = \int_{0}^{2} \frac{6}{7} \left( x^2 + \frac{xy}{2} \right) \, dy = \frac{6}{7} x^2 y + \frac{6}{7} \frac{xy^2}{4} \bigg|_0^2 = \frac{12}{7} x^2 + \frac{6}{7} x = \frac{6}{7} x (2x + 1)
\]

c) Find \( P\{X > Y\} \).

\[
P\{X > Y\} = \iiint_{\mathbb{R}} f(x, y) \, dy \, dx = \int_0^x \int_0^1 \frac{6}{7} \left( x^2 + \frac{xy}{2} \right) \, dy \, dx = \frac{6}{7} \int_0^x \left[ x^2 y + \frac{xy^2}{4} \right]_0^1 \, dx
\]

\[
= \frac{6}{7} \int_0^x x^2 + \frac{x^3}{4} \, dx = \frac{6}{7} \left[ \frac{x^3}{3} + \frac{x^4}{16} \right]_0^1 = \frac{3}{28} \cdot \frac{15}{56} = \frac{15}{56}
\]
2. (3 point) (p. 270, #4a) Suppose 3 balls are chosen with replacement from an urn consisting of 5 white balls and 8 red balls. Let $X_i = 1$ if the $i$th ball is white, 0 otherwise. Give the joint probability mass function of $X_1, X_2$.

$$p(0,0) = \left(\frac{5}{13}\right)^2$$
$$p(1,0) = p(0,1) = \left(\frac{8}{13}\right)\left(\frac{5}{13}\right)$$
$$p(1,1) = \left(\frac{8}{13}\right)^2$$

$$F = \frac{64}{16^9} + 2\left(\frac{40}{16^9}\right) + \frac{25}{16^9} = \frac{64 + 80 + 25}{16^9} = \frac{64 + 105}{16^9}$$

3. (3 point) (p. 273, #29) The gross weekly sales at a certain restaurant are a normal random variable with mean $\$2200$ and standard deviation $\$230$. What is the probability that:

a) the goal gross sales over the next 2 weeks exceeds $\$5000$?

$$X_1 = \text{week 1 sales}$$
$$\mu_1 = 2200$$
$$\sigma_1 = 220$$

$$X_2 = \text{week 2 sales}$$
$$\mu_2 = 2200$$
$$\sigma_2 = 220$$

$$X = X_1 + X_2$$
$$\mu = 4400$$
$$\sigma^2 = 105800$$
$$\sigma = 325.27$$

$$P\{X < 5000\} = P\left\{\frac{X - 4400}{325.27} < \frac{-5000 - 4400}{325.27}\right\} = \phi(-1.944) = 0.0269$$

b) weekly sales exceed $\$2000$ for in at least 2 of the next 3 weeks?

$$P\{X > 2000\} = P\left\{\frac{X - 4400}{325.27} > \frac{-2000 - 4400}{325.27}\right\} = \phi(1.87) = 0.8078$$

$$P\{\text{at least 2 of 3 weeks} = (0.8078)^2 \times 3 (0.8078)^2 (0.1928)^1 \times 0.571 + 0.3724 = 0.9045$$

$$P\{\text{all 3 weeks} = 2 of 3 \}$$