Solutions to some problems around Homework #1

January 30, 2015

#5a. The sample space $\Omega$ is the set of all pairs $(i, j)$ so that $1 \leq i, j \leq 6$. The event $E$ is $E = \{(i, j) : |i - j| \leq 1\}$. So

$$P(E) = \frac{|E|}{|\Omega|} = \frac{16}{36}.$$ 

#5b. The sample space $\Omega$ is the set of all pairs $(i, j)$ so that $1 \leq i, j \leq 6$. The event $E$ is $E = \{(i, j) : \text{either } i \geq 5 \text{ or } j \geq 5\}$. So

$$P(E) = \frac{|E|}{|\Omega|} = \frac{20}{36}.$$ 


#23. $P(A) = 1/2, P(B) = 1/2, P(A \cap B) = 1/4 = P(A)P(B)$. This shows that $A$ and $B$ are independent. To show $A$ and $C$ are independent, compute $P(C) = 1/2, P(A \cap C) = 1/4$. The fact that $B$ and $C$ are independent is similar. They are not jointly independent because $P(A \cap B \cap C) = 0$. If both the first die and the second die land on odd numbers then the sum is even.

#31. The probability of getting all As in one semester is $(1/2)^4 = 1/16$. So the probability that this doesn’t happen in one semester is $15/16$. So the probability that she does not have all As in any of 8 semesters is $(15/16)^8$. Thus the answer is $1 - (15/16)^8$. This is about .4. NOTE: this is one of those problems where it is much easier to compute the probability of the complement of the event instead of the probability of the event itself!