#31. The probability of getting all As in one semester is $(1/2)^4 = 1/16$. So the probability that this doesn’t happen in one semester is $15/16$. So the probability that she does not have all As in any of 8 semesters is $(15/16)^8$. Thus the answer is $1 - (15/16)^8$. This is about .4. NOTE: this is one of those problems where it is much easier to compute the probability of the complement of the event instead of the probability of the event itself!

#35. This number can be 0, 1, 2, 3, 4, or 5. Let us call this number $X$. Then

\[ P(X = 0) = 6/36, \quad P(X = 1) = 10/36, \quad P(X = 2) = 8/36, \quad P(X = 3) = 6/36, \quad P(X = 4) = 4/36, \quad P(X = 5) = 2/36. \]

#38. This is another one of those problems where it is easier to compute the probability of the complement of the event. Suppose they have $n$ children. The probability of them having all boys is $(1/2)^n$. The probability of all girls is also $(1/2)^n$. So the probability of at least one child of each sex is $1 - 2(1/2)^n = 1 - (1/2)^n - 1$. The answer is $n = 6$ because if $n = 6$ then the probability is at least one child of each sex is about .97. But if $n = 5$ then the probability of at least one child of each sex is about 0.94.

#42. Let $X$ be your winnings. Then

\[ \mathbb{E}[X] = 10P(X = 10) + 25P(X = 25) + 100P(X = 100). \]

Now $P(X = 100) = 1/100$, $P(X = 25) = 2/100$ and $P(X = 10) = 5/100$. So

\[ \mathbb{E}[X] = 50/100 + 50/100 + 1 = 2. \]

#47. On any single trial, the probability that 4 heads and 1 tail show up is $5/2^5 = 5/32$. Similarly, the probability that 4 tails and 1 head show up is $5/2^5 = 5/32$. So the probability of ‘success’ (meaning that someone has been determined to pay) is $10/32$. The number of trials is a geometric random variable with parameter $p = 10/32$. So the expectation is $32/10$. 

#49. Let $X$ be the number of trials. This is a geometric random variable with parameter .6. The number of minutes we will have to wait is $4(X - 1) + 20$. So the expectation is $\mathbb{E}[4(X - 1) + 20] = 4\mathbb{E}[X] - 4 + 20 = 4(10/6) + 16 = 22\frac{2}{3}$. (There is a mistake in the back of the book. Apparently, the author switched .6 with .4.

#54. First we compute the distribution of $N$. $P(N = 1) = 2/5$. $P(N = 2) = (3/5)(2/4)$. This is because there is a $3/5$ chance of not picking a defective item on the first draw. Then
if we didn’t pick a defective one on the first draw, there are only 4 left to choose from for the second draw. Similarly, \( P(N = 3) = (3/5)(2/4)(2/3) \) and \( P(N = 4) = (3/5)(2/4)(1/3) \). The expectation of \( N \) is

\[
E(N) = 1P(N = 1) + 2P(N = 2) + 3P(N = 3) + 4P(N = 4).
\]

The variance is

\[
Var(N) = E(N^2) - E(N)^2
\]

where

\[
E(N^2) = 1^2P(N = 1) + 2^2P(N = 2) + 3^2P(N = 3) + 4^2P(N = 4).
\]

#55. No. If this happened then the variance would be negative, which is impossible.

#56. For simplicity, let \( p_1 = P(X = 1) \) and \( p_2 = P(X = 2) \). Note \( P(X = 3) = 1 - p_1 - p_2 \).

So

\[
E[X] = 2.5 = p_1 + 2p_2 + 3(1 - p_1 - p_2) = 3 - 2p_1 - p_2.
\]

\[
\Rightarrow 2p_1 + p_2 = 0.5.
\]

In particular, \( p_2 = .5 - 2p_1 \). So

\[
E[X^2] = p_1 + 2^2p_2 + 3^2(1 - p_1 - p_2) = p_1 + 4(.5 - 2p_1) + 9(1 - p_1 - .5 + 2p_1)
\]

\[
= 6.5 + 2p_1.
\]

Therefore, we see that to maximize the variance, we must maximize \( p_1 \) and to minimize the variance, we must minimize \( p_1 \). However, since \( p_2 = .5 - 2p_1 \geq 0 \), \( p_1 \) cannot be larger than \( 1/4 \). So the variance is maximized when \( p_1 = 1/4, p_2 = 0 \) and \( p_3 = 3/4 \). In this case

\[
Var(X) = E(X^2) - E(X)^2 = 7 - (2.5)^2 = .75.
\]

The variance is minimized when \( p_1 = 0 \) in which case \( p_2 = .5, p_3 = .5 \) and

\[
Var(X) = E(X^2) - E(X)^2 = 6.5 - (2.5)^2 = .25.
\]

In this case the standard deviation is .5. Is this obvious without computing it?