Outstanding Loan Balances and Nonlevel Annuities

1. Outstanding Loan Balances
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Retrospective method

Assume that a loan amount $L$ is supposed to be repaid over $n$ time periods by level payments at the end of each period; we want to be able to find the balance of the loan at an intermediate time during the loan term

- $k \ldots$ denotes the time at which we want to find the loan balance just after the time $-k$ payment
- $OLB_k \ldots$ is this loan-balance, i.e.,

$$OLB_k = L \cdot a(k) - Q \cdot s_k$$

where $Q$ stands for the level amount of each of the first $k$ payments and $a$ denotes the accumulation function associated with the loan

- If $i$ is the effective interest rate per payment period in the compound interest setting, then

$$OLB_k = L(1 + i)^k - Q \cdot s_k i$$

- This method of “looking back”, i.e., of looking how much one has repaid until time $k$ when determining the loan-balance is called the retrospective method
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A loan is being repaid with 10 payments of $2,000 each followed by 10 payments of $1,000 each at the end of each half-year. Assume that the nominal rate of interest convertible semiannually equals $i^{(2)} = 10\%$.

Find the outstanding loan balance immediately after the fifth payment is made.
An Example

A loan is being repaid with 10 payments of $2,000 each followed by 10 payments of $1,000 each at the end of each half-year. Assume that the nominal rate of interest convertible semiannually equals $i^{(2)} = 10\%$.

Find the outstanding loan balance immediately after the fifth payment is made.
An Example (cont’d)

⇒ Recall the general formula:

\[
OLB_k = L(1 + i)^k - Q \cdot s_k \cdot i
\]

and let the basic time unit be a half-year.

Then, the ingredients in the above equation are:

- \(k = 5\), \(Q = 2000\), \(i = 0.05\) and the loan amount \(L\) is unknown.

We can get the loan amount \(L\) as the present value of the payments that are made to repay the loan, i.e.,

\[
L = 2000 \cdot a_{10|} + 1000 \cdot 10|_{10} a
\]

\[
= 1000(a_{20|} + a_{10|})
\]

\[
= 1000(12.4622 + 7.7217) = 20,184
\]

So,

\[
OLB_k = 20184 \cdot 1.05^5 - 2000 \cdot s_{5|0.05}
\]

\[
= 20184 \cdot 1.27628 - 2000 \cdot 5.5256 = 14,709
\]
An Example (cont’d)

⇒ Recall the general formula:

$$OLB_k = L(1 + i)^k - Q \cdot s_{\overline{k}|i}$$

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Then, the ingredients in the above equation are

$k = 5$, $Q = 2000$, $i = 0.05$ and the loan amount $L$ is unknown

We can get the loan amount $L$ as the present value of the payments that are made to repay the loan, i.e.,

$$L = 2000 \cdot a_{\overline{10}|10} + 1000 \cdot a_{\overline{10}|10}$$

$$= 1000(a_{\overline{20}|10} + a_{\overline{10}|10})$$

$$= 1000(12.4622 + 7.7217) = 20,184$$

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⇒ Recall the general formula:

\[ OLB_k = L(1 + i)^k - Q \cdot s_{\frac{k}{i}} \]

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Prospective method

• Contrary to the retrospective method, the prospective method is based on “looking into the future”, i.e., evaluating the value of remaining payments

• If the compound interest with constant rate $i$ is used, then we have that

\[ OLB_k = Q \cdot a_{n-k-1} i + R \cdot (1 + i)^{n-k} \]

where $Q$ denotes the level amount of all but the last payment and $R$ the amount in the last payment

• Naturally, if $Q = R$, the above equation is simpler

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$$OLB_k = Q \cdot a_{n-k} \cdot i$$
An Example

- A loan is being repaid with 20 annual payments of $1,000 each. At the time of the fifth payment, the borrower wishes to repay an extra $2,000 and then repay the balance using level annual payments over the next 12 years.

Of course, due to the extra $2,000 repaid at time 5, the level payments that are supposed to be made over those 12 years need to be revised. Assume that the effective rate of interest is 9%. Find the amount of the revised annual payment.

⇒ Using the prospective method, we get that the balance of the loan after the first 5 years is

\[ OLB_5 = 1000 \cdot a_{15}^1 = 1000 \cdot 8.06070 = 8060.70 \]

Thus, after the extra $2,000 are repaid, the outstanding loan balance becomes $6,060.70

Denote the revised level payment amount by \( X \). This amount should satisfy

\[ Xa_{12}^1 = 6060.70 \Rightarrow X = \frac{6060.70}{7.1607} = 846.38 \]
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An Example: Adjustable Rate Mortgage

- A borrower takes out a 30-year adjustable rate mortgage for $65,000. The interest rate for the first year is 8%. If the interest rate increases to 10% for the second year, find the increase in the monthly payments.

⇒ The monthly payment $Q$ for the first year can be found as

$$Q = \frac{65,000}{a_{360|0.08/12}} = \frac{65,000}{136.2835} = 476.95$$

So, the outstanding balance after one year is

$$OLB_{12} = 476.95a_{348|0.08/12} = 476.95 \cdot 135.1450 = 64,457.42$$

* Which method did we use above?

Then, the revised monthly payment $\tilde{Q}$ (for the second year, at least) can be obtained as

$$\tilde{Q} = \frac{64,457.42}{a_{348|0.08/12}} = \frac{64,457.42}{113.3174} = 568.82$$

Finally, the increase in the monthly payments is

$$\tilde{Q} - Q = 568.82 - 476.95 = 91.87$$
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Which method should we use?

- The answer depends on the context:
  - If the payments before time $k$ are level, while the last payments are irregular, then it is better to use the retrospective method.
  - If you do not know the total number of payments $n$, again, you use the retrospective method.
  - If the last payments are level and you know $n$, use the prospective method.

- Assignment: Examples 3.6.5, 3.6.6, 3.6.7, 3.6.9, 3.6.10
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Amount of principal paid

- The amount of principal paid in any single period \([k - 1, k]\) is
  \[OLB_{k-1} - OLB_k\]
- The amount of interest paid in any single period \([k - 1, k]\) is
  “total payment at time \(k\)”
  — “amount of principal paid during the \(k^{th}\) period”
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- The amount of interest paid in any single period $[k - 1, k]$ is 
  
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  – “amount of principal paid during the $k^{th}$ period”
An Example

- A loan is to be repaid by annual installments of $P$ at the end of each year for 10 years. You are given the following:

1. The amount of principal paid over the first 3 years is 290.35
2. The amount of principal paid over the last 3 years is 408.55

Find the total amount of interest paid during the loan term.
A loan is to be repaid by annual installments of $P$ at the end of each year for 10 years. You are given the following:

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Find the total amount of interest paid during the loan term.
An Example (cont’d)

⇒ Let \( L \) denote the loan amount, and let \( I \) denote the total amount of interest paid. Then,

\[
I = P \cdot a_{10|i} - L
\]

So, we need to find \( i, P \) and \( L \)

(1) ⇒ The amount of principal paid over the first three years is

\[
OLB_0 - OLB_3 = P \cdot (v + v^2 + \cdots + v^{10}) - P \cdot (v + v^2 + \cdots + v^7)
\]

\[
= P \cdot (v^8 + v^9 + v^{10})
\]

\[
= P \cdot v^7(v + v^2 + v^3) = 290.35
\]

(2) ⇒ The amount of principal paid over the first three years is

\[
OLB_7 - OLB_{10} = P \cdot (v + v^2 + v^3) - 0
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Dividing the above two equations, we get

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v^7 = \frac{290.35}{408.55} \Rightarrow i = 0.05
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v^7 = \frac{290.35}{408.55} \Rightarrow i = 0.05
\]
Let $L$ denote the loan amount, and let $I$ denote the total amount of interest paid. Then,

$$I = P \cdot a_{10|} i - L$$

So, we need to find $i$, $P$ and $L$.

(1) ⇒ The amount of principal paid over the first three years is

$$OLB_0 - OLB_3 = P \cdot (v + v^2 + \cdots + v^{10}) - P \cdot (v + v^2 + \cdots + v^{7})$$

$$= P \cdot (v^8 + v^9 + v^{10})$$

$$= P \cdot v^7(v + v^2 + v^3) = 290.35$$

(2) ⇒ The amount of principal paid over the first three years is

$$OLB_7 - OLB_{10} = P \cdot (v + v^2 + v^3) - 0$$

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Now, we have the interest rate \( i = 0.05 \) and we are ready to find \( P \) and \( L \).

Using condition (2) again, we get

\[
P = \frac{408.55}{v + v^2 + v^3} = \frac{408.55}{2.7234} = 150.02
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Hence,

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L = P \cdot a_{10|0.05} = 1,158.44
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Finally, the amount of interest paid is

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10 \cdot 150.02 - 1,158.44 = 341.66
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• Assignment: Calculator work from Section 3.7
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