Premium-Discount Formula and Other Bond Pricing Formulas

1. Premium-Discount Formula

2. Other Pricing Formulas for Bonds
Premium-Discount Formula and Other Bond Pricing Formulas

1. Premium-Discount Formula

2. Other Pricing Formulas for Bonds
The formula and selling at a premium

Assignment: All the examples in section 6.2!

• The premium-discount pricing formula for bonds reads as

\[ P = C(g - j)a_{m | j} + C \]

where \( C \) is the redemption amount, \( g \) is the modified coupon rate, \( j \) is the effective yield rate per coupon period, and \( n \) is the number of coupons.

• If \( P > C \), we say that the bond sells at a premium.

• The value \( P - C \) is called the premium or amount of premium for the bond, i.e.,

\[ P - C = C(g - j)a_{m | j} \]

• So, the bond sells at a premium iff \( g > j \).
The formula and selling at a premium

Assignment: All the examples in section 6.2!

- The premium-discount pricing formula for bonds reads as

\[ P = C(g - j)a_{m|j} + C \]

where \( C \) is the redemption amount, \( g \) is the modified coupon rate, \( j \) is the effective yield rate per coupon period, and \( n \) is the number of coupons.

- If \( P > C \), we say that the bond sells at a premium

- The value \( P - C \) is called the premium or amount of premium for the bond, i.e.,

\[ P - C = C(g - j)a_{m|j} \]

- So, the bond sells at a premium iff \( g > j \)
The formula and selling at a premium

Assignment: All the examples in section 6.2!

• The premium-discount pricing formula for bonds reads as

\[ P = C(g - j)a_{\overline{n}|j} + C \]

where \( C \) is the redemption amount, \( g \) is the modified coupon rate, \( j \) is the effective yield rate per coupon period, and \( n \) is the number of coupons.

• If \( P > C \), we say that the bond sells at a premium.

• The value \( P - C \) is called the premium or amount of premium for the bond, i.e.,

\[ P - C = C(g - j)a_{\overline{n}|j} \]

• So, the bond sells at a premium iff \( g > j \).
Assignment: All the examples in section 6.2!

- The premium-discount pricing formula for bonds reads as

\[ P = C(g - j)a_{m|j} + C \]

where \( C \) is the redemption amount, \( g \) is the modified coupon rate, \( j \) is the effective yield rate per coupon period, and \( n \) is the number of coupons.

- If \( P > C \), we say that the bond sells at a premium.

- The value \( P - C \) is called the premium or amount of premium for the bond, i.e.,

\[ P - C = C(g - j)a_{m|j} \]

- So, the bond sells at a premium iff \( g > j \)
Selling at a discount

• If \( P < C \), we say that the bond sells at a discount

• Then, the value \( C - P \) is called the discount or amount of discount on the bond and it equals

\[
C - P = C(j - g) a_{m,j}
\]

• So, the bond sells at a discount iff \( g < j \)
Selling at a discount

- If $P < C$, we say that the bond sells at a discount.
- Then, the value $C - P$ is called the discount or amount of discount on the bond and it equals
  \[ C - P = C(j - g)a_{mj} \]
- So, the bond sells at a discount iff $g < j$. 

Selling at a discount

- If \( P < C \), we say that the bond sells at a discount.
- Then, the value \( C - P \) is called the discount or amount of discount on the bond and it equals

\[
C - P = C(j - g)a_{\overline{m}|j}
\]

- So, the bond sells at a discount iff \( g < j \).
Selling at a discount

• If \( P < C \), we say that the bond sells at a discount

• Then, the value \( C - P \) is called the discount or amount of discount on the bond and it equals

\[
C - P = C(j - g)\overline{a_{m,j}}
\]

• So, the bond sells at a discount iff \( g < j \)
An Example

- Find the price of a $1,000 par value 10-year bond with coupons at 8.4% convertible semiannually, which will be redeemed at $1,050. The bond is bought to yield 10% convertible semiannually.

⇒ In this example, the parameters are:

\[ F = 1000 \]
\[ C = 1050 \]
\[ r = \frac{0.084}{2} = 0.042 \]
\[ g = \frac{1000}{1050} \cdot 0.042 = 0.04 \]
\[ j = \frac{0.1}{2} = 0.05 \]
\[ n = 20 \]
\[ K = 1050 \cdot 1.05^{-20} = 395.7340 \]
\[ G = \frac{0.042}{0.05} \cdot 1000 = 840 \]
An Example

- Find the price of a $1,000 par value 10-year bond with coupons at 8.4% convertible semiannually, which will be redeemed at $1,050. The bond is bought to yield 10% convertible semiannually.

⇒ In this example, the parameters are:

\[
\begin{align*}
F &= 1000 \\
C &= 1050 \\
r &= \frac{0.084}{2} = 0.042 \\
g &= \frac{1000}{1050} \cdot 0.042 = 0.04 \\
j &= \frac{0.1}{2} = 0.05 \\
n &= 20 \\
K &= 1050 \cdot 1.05^{-20} = 395.7340 \\
G &= \frac{0.042}{0.05} \cdot 1000 = 840
\end{align*}
\]
An Example (cont’d)

Using the **basic pricing formula**, we get

\[
P = Fr_a + K
\]

\[
= 1000 \cdot 0.042 \cdot a_{20|0.05} + 395.7340
\]

\[
= 42 \cdot 12.4622 + 395.7340
\]

\[
= 919.15
\]

Using the **premium-discount formula**, we get

\[
P = C + (Fr - Ci)a_n
\]

\[
= 1050 + (42 - 52.50)a_{20|0.05}
\]

\[
= 1050 + (-9.50) \cdot 12.4622
\]

\[
= 919.15
\]

Of course, the two prices are the same

- **Assignment**: Examples 6.3.5 and 6.3.6
Using the **basic pricing formula**, we get

\[
P = F r_{n\mid} + K \\
= 1000 \cdot 0.042 \cdot a_{20\mid0.05} + 395.7340 \\
= 42 \cdot 12.4622 + 395.7340 \\
= 919.15
\]

Using the **premium-discount formula**, we get

\[
P = C + (Fr - Ci)a_{n\mid} \\
= 1050 + (42 - 52.50)a_{20\mid0.05} \\
= 1050 + (-9.50) \cdot 12.4622 \\
= 919.15
\]

Of course, the two prices are the same

- **Assignment**: Examples 6.3.5 and 6.3.6
An Example (cont’d)

Using the **basic pricing formula**, we get

\[
P = F r a_{\text{m}} + K
\]

\[
= 1000 \cdot 0.042 \cdot a_{20|0.05} + 395.7340
\]

\[
= 42 \cdot 12.4622 + 395.7340
\]

\[
= 919.15
\]

Using the **premium-discount formula**, we get

\[
P = C + (F r - C i)a_{\text{m}}
\]

\[
= 1050 + (42 - 52.50)a_{20|0.05}
\]

\[
= 1050 + (-9.50) \cdot 12.4622
\]

\[
= 919.15
\]

Of course, the two prices are the same

- *Assignment:* Examples 6.3.5 and 6.3.6
Using the **basic pricing formula**, we get

\[ P = F_r a_n + K \]
\[ = 1000 \cdot 0.042 \cdot a_{20|0.05} + 395.7340 \]
\[ = 42 \cdot 12.4622 + 395.7340 \]
\[ = 919.15 \]

Using the **premium-discount formula**, we get

\[ P = C + (F_r - C_i) a_n \]
\[ = 1050 + (42 - 52.50) a_{20|0.05} \]
\[ = 1050 + (-9.50) \cdot 12.4622 \]
\[ = 919.15 \]

Of course, the two prices are the same

- **Assignment:** Examples 6.3.5 and 6.3.6
Premium-Discount Formula and Other Bond Pricing Formulas

1. Premium-Discount Formula

2. Other Pricing Formulas for Bonds
The Base Amount Formula

- If we substitute the expression for the value of the annuity in the basic formula, we get

\[ P = G - Gv_j^n + Cv_j^n = (C - G)v_j^n + G \]

where \( G \) denotes the base amount, \( v_j \) is the discount factor per coupon period and \( n \) is the number of coupons.

- The above formula is referred to as the base amount formula.
The Base Amount Formula

If we substitute the expression for the value of the annuity in the basic formula, we get

$$P = G - G v_j^n + C v_j^n = (C - G) v_j^n + G$$

where $G$ denotes the base amount, $v_j$ is the discount factor per coupon period and $n$ is the number of coupons.

The above formula is referred to as the base amount formula.
An Example (cont’d): Base amount formula

⇒ Reconsidering the earlier example, we can reevaluate the price of the bond using the base amount formula as

\[ P = G + (C - G)v^n_j \]

\[ = 840 + (1050 - 840) \left( \frac{1}{1.05} \right)^{20} \]

\[ = 840 + 210 \cdot 0.37689 \]

\[ = 919.15 \]
Makeham’s Formula

If we do not know the number of coupons $n$, but we know the present value $K$ of the redemption amount, we use Makeham’s formula:

$$P = K + \frac{g}{j} \cdot (C - K)$$

where $g$ stands for the coupon rate, $j$ is the effective yield rate per coupon period, $C$ is the redemption amount and $K$ is the present value of the redemption amount.
If we look at our example again, using Makeham’s formula, we obtain:

\[
P = K + \frac{g}{j}(C - K)
\]

\[
= 395.7340 + \frac{0.04}{0.05}(1050 - 395.7340)
\]

\[
= 919.15
\]