Provide your final answer only for the following problems:

**Problem 1.1.** (5 points) Write down the definition of the effective interest rate for the time period \([t_1, t_2]\) in terms of the accumulation function \(a(\cdot)\).

**Problem 1.2.** (5 pts) Find the total amount of interest that would be paid on a $1,000 loan over a 10−year period, if the effective interest rate is 0.09 per annum under the following repayment method:

The entire loan plus entire accumulated interest is paid as one lump-sum at the end of the loan term.

**Problem 1.3.** (5 pts) Roger initially deposits $4,000 in an investment fund which pays him $2,000 at time 1 and $4,000 at time 2. Sally gets $2,000 at time 0 and $4,000 at time 1, and deposits $5,460 at time 2 in return.

Both investments are governed by compound interest with the same annual effective interest rate \(i\) and they have the same net present values.

Find \(i\).

(a) About 9%
(b) About 10.0%
(c) About 11.5%
(d) About 12%
(e) None of the above

Please, provide your complete solutions to the following problems:

**Problem 1.4.** (5 points) Write down the definition of the (time-varying) force of interest in terms of the accumulation function \(a(\cdot)\).

**Problem 1.5.** (2 points) Assume that the force of interest is constant and denoted by \(r\). Express the accumulation function \(a(t)\) in terms of \(r\) for \(t \geq 0\).

Please, provide your final answer only to the following problems:

**Problem 1.6.** (5 pts)

Roger makes an initial deposit of \(K\) into an account governed by the time-varying continuously compounded risk-free interest rate \(r(t) = \frac{9}{10} \sqrt{t}\) (per annum).

At the same time, Harry makes an initial deposit at the same amount into an account governed by the constant annual discount rate \(d\).

There are no subsequent deposits to or withdrawals from either of the two accounts.

After 4 years, Roger and Harry realize that the balances in their accounts are equal. Which of the following is the closest to \(d\)?

(a) \(e^{-6/5}\)
(b) \(e^{-1/5}\)
(c) \(1 - e^{-1/5}\)
(d) \(1 - e^{-6/5}\)
(e) 1
Problem 1.7. (5 pts)
You invest an amount $A$ into an account at time $-0$. The account is governed by a continuously compounded risk-free interest rate equal to 0.04.
At time $-4$, you deposit an additional amount $3A$ into the account and the continuously compounded risk-free interest rate changes to 0.06.
Which of the following best describes your balance at time 8?
(a) $A(e^{0.16} + 3e^{0.24})$
(b) $A(e^{0.32} + 3e^{0.24})$
(c) $A(e^{0.40} + 3e^{0.24})$
(d) $A(e^{0.40} + 3e^{0.48})$
(e) None of the above

Example 1.1. A warm-up example
Source: “Calculus” by James Stewart.

“One model of population growth is based on the assumption that the population grows at a rate proportional to the size of the population.” Let us denote the proportionality constant by $k$ and let the function $P(\cdot)$ stand for the size of the population. Then, $P$ must satisfy the following (ordinary differential) equation:

$$\frac{dP(t)}{dt} = kP(t)$$

Let the initial population size be $p_0$. Then, the population size $P(t)$ at time $t \geq 0$ is explicitly given by:

$$P(t) = p_0e^{kt}$$

Please, provide your complete solution to the following problem:

Problem 1.8. (8 points) Continuously compounded interest
Assume that the balance in a savings account is growing so that its rate of growth is proportional to the current balance at any time. Let us denote the proportionality constant by $r$ and let the function $B(\cdot)$ stand for the balance as a function of time. Then, $B$ must satisfy the following (ordinary differential) equation:

$$\frac{dB(t)}{dt} = rB(t)$$

So, if the initial balance in the account is $b_0$, then the expression for the balance as a function of time $t \geq 0$ is

Problem 1.9. (5 points) Assume that a loan is taken out and that the loan repayment scheme is such that every installment is exactly equal to the amount of interest charged on the loan since the last payment time; in other words, every payment consists only of the interest due. What is the expression for the outstanding loan balance immediately after the $k^{th}$ payment is made?

If you believe that you have insufficient information to answer this question, just write down: “Insufficient info.”

Problem 1.10. (5 pts) Find the total amount of interest that would be paid on a $1,000 loan over a 10-year period, if the effective interest rate is 0.09 per annum under the following repayment method:

Interest is paid each year as it is accrued and the principal is repaid at the end of the loan term.