9.1. **Two binomial periods: American options.** Please, provide your complete solutions to the following problems:

**Problem 9.1.** (10 points) Find the current price of a one-year, $110-strike American put option on a non-dividend-paying stock whose current price is $S(0) = 100$. Assume that the continuously compounded interest rate equals $r = 0.06$.

Use a two-period binomial tree with $u = 1.23$, and $d = 0.86$ to calculate the price $V_P(0)$ of the put option.

**Problem 9.2.** (10 points) Solve problem #49 (p.116) from the Sample MFE Problems.

**Problem 9.3.** (20 points) Consider a two-period binomial model for a non-dividend paying asset $S$ with $S(0) = 50$ and $u = 1/d = 2$. Let $i = 0.25$ denote the effective interest rate per period. You need to price a European put option on $S$ which expires at the end of the two periods and has the strike $K = 70$.

(i) (10 pts) Find the values of the given option at all the nodes in the binomial tree. In particular, find the no-arbitrage price at time 0 of this option.

(ii) (8 pts) Find the number of shares $\Delta$ one needs to invest in at every node in the tree in order to replicate the option.

(iii) (2 pts) If the option were American, would there be early exercise?

**Problem 9.4.** (10 points)

Consider a one-period forward binomial tree with $h = 1$, $S(0) = 100$, $r = 0.08$, $\sigma = 0.3$, $\delta = 0.08$.

(a) (5 pts) Find the expression $V_{\text{call}}(0, K)$ for the time−0 price of the American call option on $S$ with strike $K$ and maturity at the end of the period.

(b) (3 pts) Determine the condition for the strike $K$ to be such that early exercise occurs?

(c) (2 pts) In particular, is there early exercise for $K = 70$?