Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points. 
Time: 50 minutes

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MULTIPLE CHOICE
1.1. DEFINITIONS.

Problem 1.1. (5 points) Write the definition of what it means for a derivative security to be long with respect to an underlying risky asset.

Solution: A derivative security is said to be long with respect to an underlying asset if its payoff is a nondecreasing function of the final price of that asset.

1.2. TRUE/FALSE QUESTIONS. Please, circle the correct answer on the front page of this exam.

Problem 1.2. (2 pts) Consider a portfolio consisting of the following four European options with the same expiration date \( T \) on the underlying asset \( S \):
- long one call with strike 40,
- long two calls with strike 50,
- short one call with strike 65.

Let \( S(T) = 69 \). Then, the payoff from the above position at time \( T \) is less than 60.

Solution: FALSE

The payoff is

\[
(69 - 40) + 2(69 - 50) - (69 - 65) = 63.
\]

Problem 1.3. A covered call consists of a written call and long underlying asset.

Solution: TRUE

Problem 1.4. It is possible for the buyer and the writer of the same option to end up having the same profit on the exercise date.

Solution: TRUE

This happens if they both break even, i.e., if both of their profits equal zero.

1.3. FREE-RESPONSE PROBLEMS.

Problem 1.5. (20 points) Which of the positions listed will benefit from the underlying asset’s price decline? Draw the payoff curves for each position and justify your answer.

(i) Short put
(ii) Long put
(iii) Short call
(iv) Short stock
(v) Short forward contract

Solution: Only the short put is long in the underlying asset.
Problem 1.6. (15 points) The time-0 spot price of the market index is $900. A 3-month
forward contract on this market index has the forward price of $930.
After 3 months the market index is priced at $920. The nominal rate of interest convertible
monthly is 4.8%.
What is the difference in the profits between a long index investment and a long forward
contract investment?

Solution: The difference between the payoffs is just the forward price of $930. Since there is
no initial cost in a forward contract, the initial cost of the entire position consist merely of the
investment in the index. It equals $900.
So, the difference between the profits is $920 − (920 − 930) − 900(1.004)^3 = 19.1567 ≈ 20.

Problem 1.7. (10 points) A strategy consists of buying a market index product at $830 and
longing a put on the index with a strike of $830. The put premium is $18.00 and interest rates
are 0.5% effective per month.
Compute the profit or loss from this position in 6 months if the market index is worth $810 at
time 6 months.

Solution: The profit from a position is defined as the position’s payoff minus the future value
of the initial cost.
If $S(T)$ denotes the price of the market index at time $T = 0.5$ (i.e., in six months), then
the payoff of the long position in the index is exactly $S(T)$. The payoff of the long put is
$(K - S(T))_+$, where $K = 830$ denotes the strike of the put. So, since $K > S(T)$, the total
payoff of this position is $S(T) + (K - S(T))_+ = S(T) + K - S(T) = K = 830$.
The initial cost of the above position is $S(0) + V_P(0)$ where $S(0)$ denotes the initial index price
and $V_P(0)$ denotes the put premium. So, the future value of the total initial cost is
$(830 + 18)(1.005)^6 = 873.76$.
The final answer is, thus, $830 - 873.76 = -43.76$.

1.4. MULTIPLE CHOICE QUESTIONS. Please, circle the correct answer on the front
page of this exam.

Problem 1.8. (5 points) The premium on a 2-month call option on the market index with an
exercise price of 1050 is $9.30 when originally purchased. After 2 months the position is closed
and the index spot price is 1072. If interest rates are 0.5% effective per month, what is the
call’s profit?
(a) $9.30
(b) $9.39
Solution: (c)
The value at expiration of the cost of the call is $9.30 \cdot 1.005^2 \approx 9.39.$
The payoff of the call is $1072 - 1050 = 22.$
So the profit is $22 - 9.39 = 12.61.$

Problem 1.9. (5 points) The spot price of the market index is $900. After 3 months the
market index is priced at $940.
An investor has a long call option on the index at a strike price of $930. After 3 months what
is the investors payoff?
(a) $10 loss
(b) $0
(c) $10 gain
(d) $20 gain
(e) None of the above.
Solution: (c)
In our usual notation, the payoff is
\[(S(T) - K)_+ = (940 - 930)_+ = 10.\]

Problem 1.10. Source: Prof. Jim Daniel (personal communication).
A stock’s price today is $1000 and the annual effective interest rate is given to be 10%. You
write a one-year, $1,050-strike call option for a premium of $10 while you simulataneously buy
the stock. What is your profit if the stock’s spot price in one year equals $1,200?
(a) $150.00
(b) $139.90
(c) $60.00
(d) - $39.00
(e) None of the above.
Solution: (d)
\[S(T) - 1000(1.10) - (S(T) - K)_+ + 10(1.10) = 1050 - 1010(1.10) = -39.\]

Problem 1.11. The initial price of the market index is $900. After 3 months the market index
is priced at $960. The effective monthly rate of interest is 1.0%.
The premium on the long put, with a strike price of $975, is $10.00. What is the profit at
expiration for this long put?
Problem 1.12. Assume that you open a 100-share short position in a common stock $S$ when the bid-ask is $100.00-$101.00. When you close your position the bid-ask prices are $99.50-$100.00. Assume that you pay a commission rate of 1.00%. Calculate your (roundtrip) gain or loss on this short investment (assume $r = 0$)?

(a) The investor breaks even; i.e., the gain/loss is 0.
(b) About $200 loss
(c) About $132.50 loss
(d) About $200 gain
(e) None of the above

Solution: (b)
The commission needs to be paid for both transactions, so the total outcome for the short-seller is

$$100(100 \cdot 0.99 - 100 \cdot 1.01) = -200.$$ 

Problem 1.13. KidCo Cereal Company sells Sugar Corns for $2.50 per box. The company will need to buy 20,000 bushels of corn in 6 months to produce 40,000 boxes of cereal. Non-corn costs total $60,000. What is the company’s profit if they purchase call options at $0.12 per bushel with a strike price of $1.60? Assume the 6-month interest rate is 4.0% effective and the spot price in 6 months is $1.65 per bushel.

(a) $11,293 loss
(b) $8,005 loss
(c) $5,504 gain
(d) $12,064 gain
(e) None of the above.
Solution: (c)
Let the price of a bushel of corn in 6 months be denoted by $S(1/2)$. Then, the answer is
$2.50 \cdot 40,000 - 20,000 \cdot S(1/2) + 20,000(S(1/12) - 1.60) - 60,000 - 20,000 \cdot 0.12 \cdot (1 + 0.04)$. 
Since it is given that $S(1/2) = 1.65$, we get
$$20,000(5 - 1.60 - 3 - 0.12 \cdot 1.04) = 5,504.$$ 

Problem 1.14. The current price of the market index is $900. The continuously compounded risk-free interest rate is 4.8%. After 3 months the market index is priced at $920. What is the profit for the writer of the $930-strike, three-month call option if the time-zero option premium equals $2.00? 
(a) About $12.02 loss 
(b) About $2.02 loss 
(c) About $2.02 gain 
(d) About $12.02 gain 
(e) None of the above.

Solution: The option is not exercised, so the profit is 
$$2e^{0.048/4} = 2.02$$

1.5. SHORT ANSWER QUESTIONS.

Problem 1.15. (10 points) From a manufacturer’s perspective, why would he/she decide to use derivative securities on their product to hedge? Respond in five lines or less.

Solution: Answers may vary, but the bottom line is that the manufacturer can prevent losses only by a limited amount using operations optimisation and other tools within his/her area of expertise. Their profit still depends heavily on market-price fluctuations — well outside of his/her area of influence and/or expertise. So, derivative securities are a welcome tool to hedge that risk.