Problem 8.1. (2 points) A down-and-in call option should always have the barrier lower than the strike price. True or false?

Solution: FALSE

Problem 8.2. (8 points)
Consider a non-dividend-paying stock whose initial price is $S(0) = 100$ and whose volatility is equal to $\sigma = 0.3$. We model the future evolution of this stock over the next year using a two-period forward binomial tree. Assume that the continuously compounded interest rate equals $r = 0.04$. Calculate the price $V_R(0)$ of a rebate option on the above stock with exercise date $T = 1$, the rebate amount equal to $R = 10$ and $K = 105$.

Solution: With the given parameters, the up and down factors equal
\[ u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{0.04/2+0.3/\sqrt{2}} = 1.2613 \quad \text{and} \quad d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{0.04/2-0.3/\sqrt{2}} = 0.8252. \]

So, we get the following tree:

We intend to use the risk-neutral pricing formula, so we need the risk-neutral probability of the stock-price going up:

\[ p^* = \frac{1}{1 + e^{\sigma\sqrt{h}}} = 0.447. \]

We look at the final payoff for the trajectories of the stock price and get:
- up-up $\Rightarrow$ $R_{dd} = 10$ with probability $(p^*)^2$,
- up-down $\Rightarrow$ $R_{ud} = 10$ with probability $p^*(1-p^*)$,
- down-up $\Rightarrow$ $R_{du} = 0$ with probability $(1-p^*)p^*$,
- down-down $\Rightarrow$ $R_{dd} = 0$ with probability $(1-p^*)^2$.
The risk-neutral pricing principle gives us the initial value of the rebate option:

\[ V_R(0) = e^{-0.04} \cdot 10 \cdot 0.447 = 4.2947. \]