Problem 5.1. (2 points) The premium on a 1000-strike, 2-month European call option on the market index is $20. After 2 months the market index spot price is 1075. If the risk-free interest rate equals 0.5% effective per month, what is the long-call profit?

Solution: In our usual notation, the profit is
\[(S(T) - K)^+ - F V_{0,T}[V_c(0)] = (1075 - 1000)^+ - 20(1.005)^2 = 54.80.\]

Problem 5.2. (3 points) The fair price today of a zero-coupon bond with redemption amount of $100 and which comes to maturity in a year is equal to $78.

You purchase an at-the-money European call option on a non-dividend paying stock whose price today is \(S(0) = 100\). The premium of this call was $10.

Write the expression for this call’s payoff, and for its profit (valued at its expiration date \(T\)) as a function of \(S(T)\) (the stock price at time \(T\)) and the time of maturity \(T\). Draw the graph of this call’s profit as a function of \(S(T)\).

Solution: From the bond price, denoting by \(r\) the annual continuously compounded interest rate, we get \(e^{-r} = 100/78\).

So, the expression for the call’s profit is
\[(S(T) - 100)^+ - 10e^{-rT} = (S(T) - 100)^+ - 10(\frac{100}{78})T.\]

Here is what the profit function looks like for \(T = 1\). For other maturities, you get the same shape.

Problem 5.3. (5 points) For what values of the final asset price is the profit of a long forward contract with the forward price \(F = 100\) and delivery date \(T\) in one year smaller than the profit of a long call on the same underlying asset with the strike price \(K = 100\) and the exercise date \(T\). Assume that the call’s premium equals $10 and that the annual effective interest rate equals 10%.

Express your answer as an interval.
Solution: The profit function of the forward contract is \( v_F(s) = s - 100 \). The profit function of the call is

\[
v_C(s) - 10 \times 1.10 = (s - 100)_+ - 11.
\]

For \( s \geq 100 \), the call’s profit is smaller than the forward contract’s profit. So, we focus on \( s < 100 \). Here we have to solve for \( s^* \) in

\[
s^* - 100 = -11 \quad \Rightarrow \quad s^* = 89.
\]

The answer is \([0, 89)\).