Properties of Asian and barrier options.

Problem 7.1. (2 points)
In our usual notation, the expression for the payoff of an average strike call option based on a geometric average is \((G(T) - S(T))_+\). True or false?

Solution: FALSE

Problem 7.2. (2 pts) The geometric average price call is always more valuable than the otherwise identical arithmetic average price call. True or false?

Solution: FALSE

Call the geometric average \(G(T)\) and the arithmetic average \(A(T)\). Due to the fact that \(G(T) \leq A(T)\), the payoffs of the two calls satisfy

\[(G(T) - K)_+ \leq (A(T) - K)_+\]

So, their original prices must be in the same ordering.

Problem 7.3. (2 pts) The premium on a standard call option and a down-and-in call are the same if the barrier price exceeds initial the stock-price. True or false?

Solution: TRUE

The down-and-in option is immediately “in”.

Problem 7.4. (4 points) Let the initial stock price equal 85, and consider an up-an-in put on this stock with barrier \(H = 80\), strike \(K = 90\) and exercise date \(T\) in one year. Denote the initial price of this barrier option by \(V_B(0)\) and let the price of an otherwise identical regular put be denoted by \(V_P(0)\). Then,

(a) \(V_B(0) = 0\)
(b) \(0 < V_B(0) < V_P(0)\)
(c) \(V_B(0) = V_P(0)\)
(d) \(V_P(0) < V_B(0)\)
(e) None of the above.

Solution: (c)

The barrier is already exceeded at time—0 implying that the option has immediately knocked-in. The barrier option is, hence, equivalent to an otherwise identical “regular” option.