Problem 6.1. (10 points) Assume that one of the no-arbitrage conditions in the binomial model for pricing options on a continuous-dividend-paying stock $S$ is violated. Namely, let

$$d < u \leq e^{(r-\delta)h}.$$ 

Construct an arbitrage portfolio and show that your proposed arbitrage portfolio is, indeed, an arbitrage portfolio.

Solution:

Let today’s stock price be denoted by $S(0)$. We short-sell one share of stock. After one period, according to the binomial model, the stock-price either rises to $S_u = uS(0)$ or drops to $S_d = dS(0)$.

Let us denote the profit of our portfolio on the second day by $X_u$ in the case the stock price went up and by $X_d$ if the stock price went down. The profit of our portfolio in those two cases are

$$X_u = e^{rh} \cdot S(0) - uS(0)e^{\delta h} = S(0)e^{\delta h}(e^{(r-\delta)h} - u) \geq 0$$

$$X_d = e^{rh} \cdot S(0) - dS(0)e^{\delta h} \geq 0 = S(0)e^{\delta h}(e^{(r-\delta)h} - d) > 0$$

We have non-negative profit in both cases and a strictly positive profit in one of the cases. Hence, the above strategy constitutes an arbitrage portfolio.