Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points.
Time: 75 minutes

MULTIPLE CHOICE

<table>
<thead>
<tr>
<th>TRUE/FALSE</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (5)</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
</tr>
<tr>
<td>2 (5)</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
</tr>
<tr>
<td>3 (5)</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
</tr>
<tr>
<td>4 (5)</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
</tr>
<tr>
<td>5 (5)</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
</tr>
<tr>
<td>6 (5)</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
</tr>
<tr>
<td>7 (5)</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
</tr>
<tr>
<td>8 (5)</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
</tr>
</tbody>
</table>
1.1. DEFINITIONS.

Problem 1.1. (5 points) Write the definition of an arbitrage portfolio.

1.2. TRUE/FALSE QUESTIONS.

Problem 1.2. (2 points) An agent is only allowed to write options on an underlying asset if he/she already owns units of the underlying.

Problem 1.3. (2 points) Derivative securities can reduce the risk of both the buyer and the writer of the security.

Problem 1.4. A covered call consists of a written call and long underlying asset.

Problem 1.5. It is possible for the buyer and the writer of the same option to end up having the same profit on the exercise date.

Problem 1.6. The prepaid forward price of non-dividend-paying stock does not depend on the delivery date.

Problem 1.7. Derivative securities are sometimes used as employee compensation.

1.3. FREE-RESPONSE PROBLEMS.

Problem 1.8. (15 points) You are a jeweler who buys silver – the primary input needed for your products. Exactly one ounce of silver is used to produce one unit of jewelry. Assume that the cost of all other inputs is negligible. You are able to sell each unit of jewelry for 200 plus 20% of the market price of silver in one year. In one year, the actual price of silver is modeled as being in one of three possible states, corresponding to the following probability table:

<table>
<thead>
<tr>
<th>Market Price of Silver in 1 year</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 = 250 ) per ounce</td>
<td>( p_1 = 0.3 )</td>
</tr>
<tr>
<td>( x_2 = 300 ) per ounce</td>
<td>( p_2 = 0.4 )</td>
</tr>
<tr>
<td>( x_3 = 350 ) per ounce</td>
<td>( p_3 = 0.3 )</td>
</tr>
</tbody>
</table>

You are considering utilizing forward contracts to lock in 1-year silver prices, in which case you would charge the customer (one year from now) 700 plus 20% of the forward price. The forward price of silver for delivery in one year is 300 per ounce. How much does your expected 1-year profit, per unit of jewelry sold, increase if you buy the silver forward?
Problem 1.9. (20 points) Suppose that the current price of a continuous-dividend-paying stock equals $100 per share. The dividend yield is given to be equal to 0.02. The continuously compounded risk-free interest rate is equal 0.05.

You observe that a forward price for delivery of this stock in one year equals $F = 105. Does this observed forward price create an arbitrage opportunity? If your answer is affirmative, propose an arbitrage portfolio and verify that your proposed portfolio is, indeed, an arbitrage portfolio. If your answer is negative, justify it!
1.4. MULTIPLE CHOICE QUESTIONS.

Please, circle the correct answer on the front page of this exam.

Problem 1.10. (5 points) You create the following portfolio of European options with the same underlying asset and the same exercise date:

(1) one long 40–strike put option,
(2) one long 50–strike call option,
(3) two written 60–strike call options.

What is the maximum possible payoff of the above portfolio?

(a) 0
(b) 10
(c) 40
(d) The payoff curve is not bounded from above.
(e) None of the above

Problem 1.11. An investor wishes to use a put option to hedge a long position in an underlying asset $S$. He is attempting to decide among otherwise identical European put options with different strikes (and all, of course, on the same underlying asset $S$). Which of the following statements is correct?

(a) Put options with higher strikes have a higher price and provide a higher floor.
(b) Put options with higher strikes have a lower price and provide a higher floor.
(c) Put options with higher strikes have a lower price and provide a lower floor.
(d) Put options with higher strikes have a higher price and provide a lower floor.
(e) None of the above.

Problem 1.12. (5 points) If your homeowner’s insurance premium is $1,000 and your deductible is $2000, what could be considered the strike price of the policy if the home has a value of $120,000?

(a) $117,000
(b) $118,000
(c) $120,000
(d) $122,000
(e) None of the above.

Problem 1.13. A stock now sells for $100 per share. The continuously compounded risk-free interest rate equals 0.05, and the continuous dividend yield equals 0.02. Find the prepaid forward price for delivery in two years.

(a) 96.08
Problem 1.14. A non-dividend-paying stock sells for $100 per share today. The continuously compounded risk-free interest rate equals 0.05. You are the seller in a one-year forward contract. Find your profit if the stock’s spot price in one year equals $130 per share.

(a) 5.13 loss
(b) 5.13 gain
(c) 24.87 loss
(d) 24.87 gain
(e) None of the above.

Problem 1.15. The future value in one year of the total costs of manufacturing a widget is $200. You will sell a widget in one year at its market price of $S(1)$. Assume that the annual effective interest rate equals 5%, and that the current price of the widget equals $230.

You now purchase a one-year, $220-strike put on one widget for a premium of $7. You sell some of the potential gain by writing a one-year, $250-strike call on one widget for a $2 premium.

What is the range of the profit of your hedged portfolio?

(a) [14.75, 44.75]
(b) [220.75, 250.75]
(c) [220, 244.75]
(d) [220, 250]
(e) None of the above.

Problem 1.16. The payoff of a cash-or-nothing call with strike price $K$ and exercise date $T$ is given by:

(a) $(S(T) - K)_+$
(b) $I_{S(T) \geq K}$
(c) $S(T)I_{S(T) \geq K}$
(d) $(S(T) - K)I_{S(T) \geq K}$
(e) None of the above.

Problem 1.17. A soy-bean farmer shorts forward contracts on soy in an amount matching his crop volume and with delivery at harvest time. Then, he is considered:
(a) an arbitrageur.
(b) a broker.
(c) a speculator.
(d) a hedger.
(e) None of the above.

**Problem 1.18.** A portfolio consists of the following:
- one long 40−strike put option,
- two long 50−strike call options,
- one short 60−strike put option.
All of the options are European, with the same underlying asset and the same exercise date. What is the portfolio’s payoff if the final price of the underlying asset equals $65?
- (a) 5
- (b) 10
- (c) 20
- (d) 25
- (e) None of the above.

**Problem 1.19.** A portfolio consists of the following:
- one short one-year, 50−strike call option with price equal to $8.50,
- one long one-year, 60−strike put option with price equal to $6.75.
All of the options are European and with the same underlying asset. Assume that the continuously compounded, risk-free interest rate equals 0.04. What is the portfolio’s profit if the final price of the underlying asset equals $55?
- (a) 1.75
- (b) 1.82
- (c) 6.82
- (d) 11.82
- (e) None of the above.