2.1. FREE-RESPONSE PROBLEMS.

Problem 2.1. (20 points) Assume that a stock pays no dividends. Its initial price is given to be $2.

Consider two European-style derivative securities on the above stock, both with the exercise date in one year. They have the following payoffs:

I: \( V_I(T) = (5S(T) - 10)_+ \)
II: \( V_{II}(T) = (10 - 5S(T))_+ \)

with \( T = 1 \). It is observed that the price of derivative I at time-0 equals \( V_I(0) = 2 \).

Given that the continuously compounded risk-free interest rate equals 0.05, what is the no-arbitrage time-0 price of derivative II?

Solution: Notice that

\[
V_I(T) = (5S(T) - 10)_+ = 5V_C(T) \\
V_{II}(T) = (10 - 5S(T))_+ = 5V_P(T),
\]

where \( V_C(T) \) and \( V_P(T) \) denote the payoffs of a 2-strike call and a 2-strike put, respectively, with the same underlying and exercise date as the two derivative securities described in the problem. We conclude that \( V_I(0) = 5V_C(0) \) and \( V_{II}(0) = 5V_P(0) \), in our usual notation.

On the other hand, put-call parity gives us

\[
V_C(0) - V_P(0) = S(0) - Ke^{-rT} \quad \Rightarrow \quad V_P(0) = V_C(0) - S(0) + Ke^{-rT}.
\]

Finally,

\[
V_{II}(0) = 5(V_C(0) - S(0) + Ke^{-rT}) = 5V_C(0) - 5S(0) + 5Ke^{-rT} = 2 - 10 + 10e^{-0.05} \approx 1.51.
\]
2.2. MULTIPLE CHOICE QUESTIONS. Please, circle the correct answer on the front page of this exam.

Problem 2.2. Please, solve the Sample FM(DM) Problem #50.
Solution: (b)

Problem 2.3. An investor bought a six-month, (70, 80)-put bull spread on an index. The $70-strike, six-month put is currently valued at $1, while the $80-strike, six-month put is currently valued at $8.
Assume that the continuously-compounded, risk-free interest rate equals 0.02.
What is the break-even final index price for the above put bull spread?
(a) $62.86
(b) $71.84
(c) $72.86
(d) $73
(e) None of the above.
Solution: (c)
We need to solve for $s$ such that $70 < s < 80$, in
$$80 - s = (8 - 1)e^{0.02} \Rightarrow s = 72.8586.$$

Problem 2.4. (5 points) An investor wants to hold 100 euros two years from today. The spot exchange rate is $1.37 per euro. If the euro-denominated continuously compounded annual interest rate equals 3.0% what is the price of a currency prepaid forward (rounded to the nearest dollar) ?
(a) 129
(b) 176
(c) 200
(d) 247
(e) None of the above.
Solution: (a)
$$F_{0,T}^P(x) = 100e^{-0.03 \cdot 2} \cdot 1.37 = 129.02$$

2.3. TRUE/FALSE QUESTIONS.

Problem 2.5. (2 pts) Consider a European gap put option such that its trigger price exceeds its strike price. Then, the premium of this option is decreasing with respect to the trigger price.
Solution: TRUE
Let us look at the payoff of this option at time $T$; I am adding the trigger price $K_2$ in the notation to emphasise that we are considering it to be the argument of the payoff function.

$$V_{GP}(T, K_2) = (K_1 - S(T))\mathbb{I}_{[S(T)<K_2]}.$$  

Since we are given that $K_1 < K_2$, the above payoff is negative for all the values of $S(T)$ such that $K_1 < S(T) < K_2$. Keeping all else fixed, and increasing the value of $K_2$, we see that the above region becomes “wider-and-wider”. It is evident that for all else kept intact, i.e., temporarily fixing $K_1, S(T)$, and $T$, the function $V_{GP}$ is decreasing as a function of $K_2$. We have to conclude that this effect is reflected in the initial premium as well.

**Problem 2.6.** (2 pts) Consider a gap option whose trigger price is equal to its strike price. Then, the premium for this option is the same as that for an ordinary option with the same strike, the same exercise date and the same underlying asset.

**Solution:** TRUE

**Problem 2.7.** (2 pts) The prices of the European call and put options on the same futures contract with the same exercise date are the same if and only if both options are at-the-money.

**Solution:** TRUE
One can simply use put-call parity (as we did in class).

**Problem 2.8.** (2 points) The payoff of a chooser option with the choice date coinciding with the exercise date $T$ and with the strike $K$ is given as $|S(T) - K|$.

**Solution:** TRUE

**Problem 2.9.** (2 points) Strangles are financial positions designed to hedge against increasing prices of the underlying asset.

**Solution:** FALSE

**Problem 2.10.** (2 points) The payoff of a gap put option is always nonnegative regardless of the choice of the trigger and the strike.

**Solution:** FALSE