Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points.

Time: 50 minutes

TRUE/FALSE

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FOR GRADER’S USE ONLY:

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2.1. THE DEFINITION.

Problem 2.1. (5 points)
Write the definition of an replicating portfolio of a European-style derivative security.

Solution:
A financial portfolio is said to be a replicating portfolio of a European-style derivative security if their payoffs are equal in all states-of-the-world.

2.2. TRUE/FALSE QUESTIONS. Please, circle the correct answer on the front page of this exam.

Problem 2.2. The expiration date of a futures option cannot exceed the delivery date of the underlying futures contract.
Solution: TRUE

Problem 2.3. (2 points) Exchange options are options where the underlying asset is an exchange rate.
Solution: FALSE

Problem 2.4. (2 points) A box spread is a replicating portfolio for a bond.
Solution: TRUE

Problem 2.5. (2 points) The payoff function of a ratio spread is never bounded from above.
Solution: FALSE

Problem 2.6. (2 points) A straddle has a nonnegative payoff function.
Solution: TRUE
2.3. **FREE-RESPONSE PROBLEMS.**

**Problem 2.7.** (25 points) You produce cupcakes. You plan to sell 1,000 festive cupcakes in a month. Your (unhedged) payoff will be $25,000 − S(1)$, where $S(1)$ denotes the price of the 1,000 fondant reindeer required to decorate the 1,000 cakes.

Assume that the continuously compounded annual risk-free interest rate equals 6%.

Your hedge consists of the following two components:

1. one long one-month, $10,000-strike call option on the fondant reindeer you need; its premium is $V_C(0) = $70.00,
2. one written one-month, $9,000-strike put option on the fondant reindeer you need; its premium is $V_P(0) = $150.00.

Calculate the maximum and the minimum of the profit for the (overall) **hedged** portfolio.

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**Solution:** The hedged portfolio consists of the following components:

1. the **payoff** from the cupcake sales,
2. one long one-month, $10,000-strike call option on the fondant reindeer whose premium was $70.00,
3. one written one-month, $9,000-strike put option on the fondant reindeer whose premium was $150.00.

The initial cost for this portfolio is the cost of hedging (all other accumulated production costs are incorporated in the revenue expression $25,000 − S(1)$). Their future value is

$$(70 − 150)e^{0.065} \approx −80.401.$$

As usual, the negative initial cost signifies an initial influx of money for the principal character. The **hedged** profit is

$$25000 − S(1) − (9000 − S(1))_+ + (S(1) − 10000)_+ + 80.401.$$

The minimum is attained for $S(1) \leq 9,000$. It equals

$$16000 + 80.401 = 16,080.401.$$

The maximum profit is attained for $S(1) \geq 10,000$. It equals

$$15000 + 80.401 = 15,080.401.$$
Problem 2.8. (25 points) Source: Sample FM(DM) Problem #40.

An investor is analyzing the costs of two-year, European options for aluminum and zinc at a particular strike price.

For each ton of aluminum, the two-year forward price is 1400, a call option costs 700, and a put option costs 550.

For each ton of zinc, the two-year forward price is 1600 and a put option costs 550.

The risk-free annual effective interest rate is a constant 6%. Calculate the cost of a call option per ton of zinc.

Solution:
The prices of derivatives on aluminum can be used to determine the identical strike price the call and put options share in this problem. By put-call parity, we have

\[ V_{C,Al}(0) - V_{P,Al}(0) = F_{0,T}(Al) - K(1+i)^{-T} \]

with \( i = 0.06 \). Equivalently, we can write

\[ (1+i)^2(V_{C,Al}(0) - V_{P,Al}(0)) = F_{0,T}(Al) - K. \]

Hence,

\[ K = 1400 - (1.06)^2(700 - 550) = 1231.46. \]

Using put-call parity once again – this time on derivative securities with zinc as the underlying – we obtain,

\[ V_{C,Zn}(0) = V_{P,Zn}(0) + (F_{0,T}(Zn) - K)(1+i)^{-T} = 550 + 328 = 878. \]

More compactly, we could have combined the two calculations above into

\[ V_{C,Zn}(0) = V_{P,Zn}(0) + (F_{0,T}(Zn) - F_{0,T}(Al)) + (1+i)(V_{C,Al}(0) - V_{P,Al}(0))(1+i)^{-T} \]

\[ = V_{P,Zn}(0) + (F_{0,T}(Zn) - F_{0,T}(Al))(1+i)^{-T} + (V_{C,Al}(0) - V_{P,Al}(0)) \]

\[ = 550 + (1600 - 1400)(1.06)^{-2} + (700 - 550) = 878. \]

The price \( V_{C,Zn}(0) \) of the two-year call on zinc equals:
MULTIPLE CHOICE QUESTIONS

Please, circle the correct answer on the front page of this exam.

Problem 2.9. An investor bought a six-month, (70, 80)-put bull spread on an index. The $70-strike, six-month put is currently valued at $1, while the $80-strike, six-month put is currently valued at $8. Assume that the continuously-compounded, risk-free interest rate equals 0.02.

What is the break-even final index price for the above put bull spread?

(a) $62.86
(b) $71.84
(c) $72.86
(d) $73
(e) None of the above.

Solution: (c), (d) or (e)
We need to solve for \( s \) such that 70 < \( s \) < 80, in

\[
80 - s = (8 - 1)e^{0.01} \quad \Rightarrow \quad s = 72.93
\]

Problem 2.10. (5 points) Assume that the current exchange rate is $1.3 per euro. The continuously compounded interest rate for the euro is 0.03, while continuously compounded interest rate for the USD is 0.04.

Let the price of an at-the-money USD-denominated European call on on the euro with exercise date in 6 months be equal to 0.053

What is the price of an at-the-money Euro-denominated put on the USD with the exercise date in 6 months ?

(a) About 0.011.
(b) About 0.031.
(c) About 0.051.
(d) About 0.071.
(e) None of the above

Solution: (b)
Let \( x \) denote the exchange rate from euros to dollars. We are given that \( x(0) = 1.3 \). Using the put-call duality for options on currencies, we get

\[
V_{\text{Euro}}^P(0, 1/x(0)) = (1/x(0))^2 V_{\text{USD}}^C(0, x(0)) \approx 0.031.
\]

Problem 2.11. You construct an asymmetric butterfly spread using the following three types of European options on the same asset and with the same exercise date:

- a $50-strike call,
- a $60-strike call,
- a $65-strike call.

You are told that there is exactly one short $60-strike call in the asymmetric butterfly spread. What is the maximal payoff of the above butterfly spread?

(a) 0
(b) 10/3
(c) 5
(d) The payoff is not bounded from above.
(e) None of the above.

Solution: (b)
With the given strike prices, the asymmetric butterfly spread consists of the following components:

- 1/3 of a long $50-strike call,
- one short $60-strike call, and
- 2/3 of a long $65-strike call.

The maximal payoff is attained for the final stock price equal to the “inner” strike of $60. We get

\[ \frac{1}{3} (60 - 50)_{+} - (60 - 60)_{+} + \frac{2}{3} (60 - 65)_{+} = \frac{10}{3}. \]
**Problem 2.12.** The current price of stock $S$ is $50. Stock $S$ is scheduled to pay a $3$-dividend in two months.

The current price of stock $Q$ is $60. Stock $Q$ is scheduled to pay dividends continuously with the dividend yield $0.03$.

A six-month European exchange call option with underlying asset $S$ and the strike asset $Q$ is sold for $2.75$.

The continuously-compounded, risk-free interest rate is given to be $0.04$.

What is the price of the six-month European exchange put option with underlying asset $S$ and the strike asset $Q$?

(a) About $8.58$
(b) About $9.04$
(c) About $12.75$
(d) About $14.54$
(e) None of the above.

**Solution:** (d)

$$V_{EP}(0, S, Q) = V_{EC}(0, S, Q) + F_{0,T}^P(Q) - F_{0,T}^P(S) = 2.75 + 60e^{-0.04 \cdot 0.5} - 50 + 3e^{-0.04/6} = 14.54.$$ 

**Problem 2.13.** The following two one-year European put options on the same asset are available in the market:

- a $50$-strike put with the premium of $5$,
- a $55$-strike put with the premium of $10$.

The continuously compounded, risk-free interest rate is $0.04$.

Which of the following positions certainly exploits the arbitrage opportunity caused by the above put premia?

(a) Put bull spread.
(b) Put bear spread.
(c) Both of the above positions.
(d) There is no arbitrage opportunity.
(e) None of the above.

**Solution:** (a)

**Problem 2.14.** A long strangle position...

(a) is equivalent to a short ratio spread.
(b) can be replicated with a short call and a long put with the same strike, underlying asset and exercise date.
(c) is always strictly more expensive than the straddle on the same underlying asset and with the same exercise date.
(d) is a speculation on the stock’s volatility.
(e) None of the above.

**Solution:** (d)
Problem 2.15. A stock is currently priced at $100 per share. It is scheduled to pay a continuous dividend in the amount proportional to its price with the dividend yield of 1.5%. One-year, $102-strike European call and put options on this stock have equal prices. Let the continuously-compounded annual risk-free rate of interest be denoted by $r$. Then,

(a) $r \leq 0.04$
(b) $0.04 < r \leq 0.045$
(c) $0.045 < r \leq 0.05$
(d) $0.05 < r \leq 0.055$
(e) None of the above.

Solution: (a)
By the put-call parity, we have

$$K = F_{0,T}(S) = S(0)e^{(r-\delta)T} \Rightarrow r = \delta + \frac{1}{T} \ln \left( \frac{K}{S(0)} \right).$$

So,

$$r = \delta + \frac{1}{T} \ln \left( \frac{K}{S(0)} \right) = 0.015 + \ln(102/100) = 0.0348.$$

Problem 2.16. You are given that the price of:

- a $50-strike, one-year European call equals $8,
- a $65-strike, one-year European call equals $2.

Both options have the same underlying asset. What is the maximal price of a $56-strike, one-year European call such that there is no arbitrage in our market model?

(a) $4.40$
(b) $5$
(c) $5.60$
(d) $6.02$
(e) None of the above.

Solution: (c)
Using the convexity of call price with respect to the strike, we get the following answer:

$$\frac{3}{5} \times 8 + \frac{2}{5} \times 2 = \frac{24 + 4}{5} = 5.60.$$