Put-call parity: The general case

6.1. Construction. So far, we have looked at put-call parity for non-dividend-paying assets. Now, we will use a similar approach to obtain put-call parity for stocks that pay either discrete dividends, or a continuous dividend stream.

Let Portfolio A consist of a long European call and a short European put on the same underlying asset \( S \) with the same strike \( K \) and the same exercise date \( T \). The initial value of this portfolio is

\[
V_A(0) = V_C(0) - V_P(0).
\]

There are no intermediate cash-flows associated with this portfolio and its payoff at time \( T \) is

\[
V_C(T) - V_P(T) = S(T) - K.
\]

On the other hand, let Portfolio B consist of the following:

1. a long prepaid forward contract on \( S \) for delivery at time \( T \),
2. borrowing the present value of the strike price to be repaid at time \( T \).

Then, the initial cost of this portfolio equals:

\[
F^P_{0,T}(S) - PV_{0,T}(K).
\]

Since there are no intermediate cash-flows associated with this portfolio, either, its payoff at time \( T \) is

\[
S(T) - K.
\]

Since the above portfolios have the same final payoff, by the no-arbitrage principle, we conclude that their initial values must also be the same. We get the more general version of put-call parity:

\[
V_C(0) - V_P(0) = F^P_{0,T}(S) - PV_{0,T}(K).
\]

6.2. Special cases. Our most common setting is the one with a continuously compounded interest rate \( r \). In that case the put-call parity reads as

\[
V_C(0) - V_P(0) = F^P_{0,T}(S) - Ke^{-rT}.
\]

With respect to dividends, these are the three cases we will be looking into:

- non-dividend-paying stocks:
  \[
  V_C(0) - V_P(0) = S(0) - Ke^{-rT}
  \]
- discrete dividends \( D_i, i = 1, \ldots, n \) at times \( 0 < t_1 < \cdots < t_n \leq T \):
  \[
  V_C(0) - V_P(0) = S(0) - \sum_{i=1}^{n} D_i e^{-r t_i} - Ke^{-rT}
  \]
• continuous dividends at the rate $\delta$:

$$V_C(0) - V_P(0) = S(0)e^{-\delta T} - Ke^{-rT}$$

6.3. MFE Exam Spring 2007: Problem #1. On April 30, 2007, a common stock is priced at $52.00. You are given that:

1. Dividends in equal amounts are to be paid on June 30, 2007, and on September 30, 2007.
2. A European call on the above stock with strike $K = 50$ and the exercise date in six months sells for $4.50$.
3. A European put on the above stock with strike $K = 50$ and the exercise date in six months sells for $2.45$.
4. The continuously-compounded risk-free interest rate equals 0.06.

Calculate the amount of each dividend.

Solution. In addition to our usual notation, we introduce $D$ to stand for the amount of each dividend payment. Then, the put-call parity reads as

$$V_C(0) - V_P(0) = S(0) - De^{-rt_1} - De^{-rt_2} - Ke^{-rT}$$

with $t_1 = 1/6$ and $t_2 = 5/12$. Solving for $D$ above, we get

$$D = \frac{S(0) - Ke^{-rT} - V_C(0) + V_P(0)}{e^{-rt_1} + e^{-rt_2}} = \frac{52 - 50e^{-0.06(1/2)} - 4.5 + 2.45}{e^{-0.06(1/6)} + e^{-0.06(5/12)}} \approx 0.73.$$