Provide your complete solutions for the following problems.

**Problem 1.1.** (5 points) Harry takes out a 5-year loan for $1,000. Assume that he repays the loan with end-of-year, interest-only payments and a final payment of $1,000 at the end of year 5.

The effective interest charged on Harry’s loan is time-varying, but deterministic. More precisely, it is given to be

\[ i_k = 0.07 - 0.01k \]

for year \( k \) with \( k = 1, \ldots, 5 \).

What are the payment amounts paid at the end of every year?

**Solution:**

Year 1: \( 1,000i_1 = 1,000(0.07 - 0.01) = 60 \)

Year 2: \( 1,000i_2 = 1,000(0.07 - 0.02) = 50 \)

Year 3: \( 1,000i_3 = 1,000(0.07 - 0.03) = 40 \)

Year 4: \( 1,000i_4 = 1,000(0.07 - 0.04) = 30 \)

Year 5: \( 1,000i_5 + 1,000 = 1,000(0.07 - 0.05) + 1,000 = 1,020 \)

**Problem 1.2.** (5 points)

Harry takes a $1,000, 2-year loan from Roger. Assume that he repays the loan with end-of-year, interest-only payments and a final payment of $1,000 at the end of year 2.

Neither Roger, nor Harry is particularly risk-averse, so they decide to define the interest charged in the following way:

At time 0, they toss a fair coin. If the result of the coin toss is *Heads*, the effective interest rate charged for the first year is 0.03. If the result of the coin toss is *Tails*, the effective interest rate charged for the first year is 0.04.

At time 1, they roll a fair die. If the result is a *divisible by three*, the effective interest rate charged for the second year is 0.04. Otherwise, the effective interest rate charged for the second year is 0.01.

What is the expected amount of interest Harry pays according to the above scheme?

**Solution:**

\[
1,000 \left[ \frac{1}{2} (0.03 + 0.04) + \frac{1}{3} (0.04 + 2 \times 0.01) \right] = 1,000(0.035 + 0.02) = 55.
\]