SIR Model of Epidemic

- Infective: Has the disease and can infect others
- Susceptible: Can contract the disease
- Removed: Former infectives who are no longer infectious (includes recovered, died, isolated)
- \( S = \) number of susceptibles, \( I = \) number of infectives, \( R = \) number removed
- Assumptions:
  - \( S + I + R = N \) constant
  - Incubation time negligible

Model Equations

- \( \frac{dS}{dt} = - \beta SI \)
- \( \frac{dI}{dt} = \beta SI - \gamma I \)
- \( \frac{dR}{dt} = \gamma I \)
- \( \text{infection rate} = \beta \)
- \( \text{removal rate} = \gamma \) (inversely proportional to infectious period)
- These equations say …

Consequences

- \( S = S_0 \exp(- (1/ \rho)(R - R_0)) \), where \( \rho = \gamma / \beta = \text{relative removal rate} \)
- Threshold Theorem: If \( S_0 < \rho \), then the infection dies out (no epidemic). If \( S_0 > \rho \), then the disease spreads (there is an epidemic). \( (S_0 / \rho = \text{basic reproduction rate}, \text{confusingly denoted } R_0) \)
- The maximum value of \( I \) is \( S_0 + I_0 + \rho \ln(\rho / (eS_0)) \)

Modified SIR Models

- Recurrent diseases: Infectives or removes can become susceptible
- Recruitment: e.g., new births in modeling measles
- Vaccination: (susceptibles become immune)
- Latent class: Infected but not yet infectious (SEIR)
- Maternal antibody class
- Interacting classes (e.g., STD’s, malaria)
- Variable incubation period (e.g., AIDS)