Homework 8 M340 L

The homework is due on Tuesday, November 11th, before the lecture.

Problem 1: To warm up!
1) Section 5.1, problems 21 and 22.
2) Section 5.2 problems 21 and 22.

Problem 2:
1) Section 5.1, problems 2, 4, 6 and 8.
2) Section 5.1, problems 10, 12, 14 and 18.

Problem 3:
1) Section 5.1 problems 24, 25, 27.
2) Section 5.1, problems 31, 32, 35.

Problem 4:
1) Section 5.2, problems 6 and 8.
2) Section 5.2, problems 10, 12, 16 and 17.

Problem 5:
1) Section 5.2, problem 18.
2) We consider the linear transformation of counter-clockwise rotation through an angle $\theta$. Write the standard matrix $A$. Find the eigenvalues and the eigenvectors of $A$. What does it mean geometrically?
3) Consider the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix}$. This transformation corresponds to a rotation about the x-axis by an angle $\theta$. What are the eigenvectors of $A$ to the eigenvalue 1. What does it mean geometrically?

Problem 6:
1) Section 5.3, problems 2 and 4.
2) Section 5.3, problems 10, 14, 18.

Problem 7:
1) Let $A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$. Show that $A$ has no real eigenvalues.
   Extra credit +4pts: What are the complex eigenvalues? What are the corresponding eigenvectors?
2) Write the standard matrix of the reflection through the line $y=x$. What are the eigenvalues of this matrix? What are its eigenvectors? What does it mean geometrically?
3) Let $A$ be a $2 \times 2$-matrix. Show that the sum of the diagonal entries equals the sum of the eigenvalues.
Problem 8:

1) Let $A$, $U$ and $B$ be $n \times n$-matrices such that $A = UBU^{-1}$. Show that $A$ and $B$ have the same determinant. What about their characteristic polynomial?

2) Let $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$. Find the eigenvalues and eigenvectors of $A$, $A^2$, $A^k$, $A + 4I$. What do you notice?

(Extra credit +5pts: Generalize your observations to any $2 \times 2$ matrix.)