Lecture 1

Website: math.utexas.edu/~schollcr/341
Syllabus posted there (/users/schollcr ?)
(bring copies next time)
HW due Thursdays in class
(starting next week)

Office hours: Tuesdays 3:30-5. RLM 12.154
(if you can't make it then email me).

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Pick up course textbook!
Follow along: read section in advance before class.

Studying: §1.1 today + Thursday
then start on §1.2.

Usual grading policies: see syllabus.

Autotune: force you to sing in tune:
it corrects your pitch by modifying the soundtrack.
It picks closest note and corrects your pitch to that.
I.e. the closest piano key note to what you are singing.

What happens: you sing and this produces sound waves (i.e. change in air pressure)

Can graph it: get raw wave file (~.wav)

$y = \text{pressure}$

\[ \text{time} = t \]

The higher the note the more rapid the oscillation.

Idealized note: pure sinusoidal

\[ y = A \cdot \sin(k t + \phi) \]

- $A$ = amplitude
- $k$ = frequency

E.g. 440 Hz = 440 oscillations/see

- A above middle C = double frequency
- 880 Hz: next higher A.

Autotune: essentially finds the strongest frequencies.
This course: Linear algebra and matrix theory.

The job of finding the strongest frequency is done by Linear Algebra.

Fourier Transform: It replaces time with frequency.

Given a sound wave \( f(t) \) \( t = \text{time} \)

Get: \( \hat{f}(f) \) \( \text{Fourier transform} \)

Given more complicated signal: \( \hat{f}(f) \) will have peaks at frequencies such that \( f \) is sum of pure harmonic of these frequencies.
\[ f(t) = \text{sum} \, A \sin(440t + c_1) + B \sin(880t + c_2) \]

Linear algebra provides \( F \).

**Key idea:** \( F \) expresses \( f \) in the basis of pure frequencies. \( F(f) \) tells you the frequencies.

\( F(f) \) is telling you how to express \( f \) in the basis of pure tones.

**Why "Linear."** Why is Fourier transform linear?

Linear function: \( y = mx + b \).

For us, \( b = 0 \): \( y = mx \). Linear \( \mathbb{R} \to \mathbb{R} \)

\( x \mapsto y = mx \).

Linear function of two variables:
\( g(u,v) = a.u + b.v \), \( a,b \in \mathbb{R} \).

No quadratic.
MVC is about approximating all functions by linear ones. You do it by taking derivative. I.e. calculus (via first deriv.) reduces everything to lin alg.

Example: One variable: \( f: \mathbb{R} \to \mathbb{R} \)

\[
f(x) \approx f(x_0) + f'(x_0)(x-x_0)
\]

Near \( x_0 \) slope \( f'(x_0) \)
RHS = tangent line to \( f \) at \( x_0 \)

Two variables: \( f(x, y) \approx f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y-y_0) \)

RHS = linear approximation to \( f \) near \( (x_0, y_0, f(x_0, y_0)) \)

Linear because of form \( ax + by + c \).

Linear algebra: we generalize this to vectors.

Definition A linear function \( f \) is one satisfying

\[
\begin{align*}
  f(u+v) &= f(u) + f(v), \\
  f(au) &= a f(u), \\
  f(0) &= 0
\end{align*}
\]
Linear functions in one variable:
\[ f(x) = mx \quad m \in \mathbb{R} \]

Linear in two variables:
\[ f(x,y) = ax + by \quad a, b \in \mathbb{R} \]

In terms of what you are used to, this definition is: linear with no constant term.

(If \( f(x,y) = ax + by + c \), then \( f(0,0) = c \), but we need \( f(0,0) = 0 \) by definition.)

Note: \( (x,y) = (ax, ay) \) so part 2 of definition of linear says in particular \( f(0,0) = F(0, (0,y)) = 0 \cdot f(0,y) = 0 \).

Observation: Fourier transform is linear:

a) \( F(f + g) = F(f) + F(g) \).

b) \( F(af) = a \cdot F(f) \).

a): If you take \( f = \text{side of freq } 440 \)
\( g = \text{side of freq } 880 \)
\[ F(f + g) = \begin{bmatrix} f(e) & f(g) \\ 440 & 880 \end{bmatrix} = F(f) + F(g) \]

Note: \( f, g \) are functions: the space of functions has infinitely many variables.
\[ \text{(infinite many times).} \]

Not on exam: this is motivation.

End of intro.

Plan for course: slowly introduce linear algebra, via careful definitions especially: (large \( n \)): Fourier transform eventually.

In particular: \( n = \text{number of samples} = \text{sampling rate (per second)} = \text{time (in seconds)} \)

§1.1. Fundamental operations with vectors.

[see later: vectors are the domain of linear functions].

\( \mathbb{R} = \text{all real numbers} \), i.e. coordinate values along real line.

Definition A real \( n \)-vector is a sequence of \( n \) real numbers.

aka: an (ordered) \( n \)-tuple of real numbers:

i.e.: \( [a_1, a_2, \ldots, a_n] \), \( a_1, a_2, \ldots, a_n \) are in \( \mathbb{R} \).

\( a_1, a_2, \ldots, a_n \in \mathbb{R} \).

Example: \( \mathbb{R}^2 = \text{set of real 2-vectors ("2-dim space") (n=2)} \)

example: \( [2, 47], [0, 92] \)
The set of real n-vectors is denoted \( \mathbb{R}^n \).

**Definition** The zero vector: 
\[ [0, 0, \ldots, 0] \] is the vector whose coordinates are all zero.

**Definition** Two vectors are equal if 
\[ (a_1, a_2, \ldots, a_n) = (b_1, b_2, \ldots, b_n) \] if and only if 
\[ a_1 = b_1, \quad a_2 = b_2, \ldots, \quad a_n = b_n. \]

**Notation:** A real \( c \in \mathbb{R} \) is called a "scalar" distinguished from \( n \)-vectors: a scalar is NOT a vector.

\[ \text{I.e.} \quad c = \begin{bmatrix} c \end{bmatrix}. \]

**Geometric interpretation:**

\[ (a, b) \quad \text{and} \quad (c+1, b) \]

\[ v = \begin{bmatrix} a \end{bmatrix}, \quad w = \begin{bmatrix} c \end{bmatrix}, \quad v + w = \begin{bmatrix} a + c \end{bmatrix}. \]

**Example:**

\[ v = \begin{bmatrix} 2 \end{bmatrix}, \quad w = \begin{bmatrix} 1 \end{bmatrix} \]

\[ v \pm w. \]
I.e. a vector is a direction and a length. (\textbf{Vill})

Length of a two-vector: \( \mathbf{v} = [a, b] \)
Length: \( ||\mathbf{v}|| = \sqrt{a^2+b^2} \). (Pythagorean theorem)

\[
\begin{bmatrix}
a_b \\
b \\
a_0 \\
\end{bmatrix}
\Rightarrow
|\begin{bmatrix}a_b \\
b \\
a_0 \\
\end{bmatrix}| = \sqrt{a^2+b^2}.\
\]

3-vectors: \( \mathbf{v} = [a, b, c] \)

\[
\begin{aligned}
u & = [a, b, 0] \\
w & = [0, 0, c]
\end{aligned}
\]

\[||u|| = \sqrt{a^2+b^2}
\]
\[||w|| = c \quad \text{(if } c \text{ positive)}
\]

\[||\mathbf{v}|| = \sqrt{(\sqrt{a^2+b^2})^2+c^2} = \sqrt{a^2+b^2+c^2}.
\]

\textbf{Definition} The length, \( ||\mathbf{a}|| \), of a vector \( \mathbf{a} = [a_1, a_2, \ldots, a_n] \) is:

\[||\mathbf{a}|| = \sqrt{a_1^2+a_2^2+\ldots+a_n^2}.
\]

(\(||\mathbf{a}||\) is the notation for "length" or "magnitude" of \( \mathbf{a} \).)

\textbf{Ex.}: \( ||[3, 4]| | = \sqrt{3^2+4^2} = 5
\)
\(||[5, 12]| | = \sqrt{5^2+12^2} = 13 \Rightarrow ||[3, 9, 12]| | = 13.\)
\[ \| (3, 4, 12) \| = \sqrt{3^2 + 4^2 + 12^2} = 13 \]  
\[ \text{ex} \quad \| (1, 2, 0, 3) \| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14} \]

**Definition**  The sum of two vectors is the sum of the coordinates:

\[ [a_1, a_2, \ldots, a_n] + [b_1, b_2, \ldots, b_n] \]

\[ := [a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n] \]

defined as.

\[ \text{i.e. means "equal because we define it that way." } \]

**Definition:** Scalar multiplication:

\[ c \begin{bmatrix} a_1, a_2, \ldots, a_n \end{bmatrix} \]

\[ = \begin{bmatrix} ca_1, ca_2, \ldots, ca_n \end{bmatrix} \quad | \quad c \in \mathbb{R} \]

Scalar \cdot Vector

Geometric interpretation:

\[ \text{e.g. } v = (0, 1) \quad u = (0, 0, 3) \quad w = (0, 0, b) \]

\[ v = u + w \]
Scalar mult: keeps or reverses direction. Modifies length.

$2 \cdot \mathbf{v} = \text{vector length twice that of } \mathbf{v}.$

Same direction.

$2[\mathbf{v}] = (2a, 2b)$

(\vec{a}, \vec{b}) \rightarrow (2\vec{a}, 2\vec{b})$