The Proof of Proposition 5.3.2 in Dr. Shirley's Format.

Proposition 5.3.2: For all integers \( n \geq 3 \), \( 2n+1 < 2^n \).

Proof: [By Mathematical Induction]

Let \( n = 3 \).

\[ 2n+1 = 2 \times 3 + 1 = 7, \text{ by substitution} \]
\[ 2^n = 2^3 = 8, \text{ by substitution} \]
\[ 7 < 8. \]

[End of Basis Step]

\[ \vdash \text{For } n = 3, \quad 2n+1 < 2^n, \text{ by substitution}. \]

Now, suppose that \( k \) is any integer with \( k \geq 3 \).

Suppose that \( 2k+1 < 2^k \). [The Inductive Hypothesis]

[\text{NTS: } 2(k+1)+1 < 2^{(k+1)}]

\[ \vdash 2(k+1)+1 = 2k+2+1 \]
\[ = (2k+1)+2 \]

Since \( k \geq 3 \), \( 2 < 2^k \).

By the Inductive Hypothesis, \( 2k+1 < 2^k \).

\[ \vdash (2k+1)+2 < 2^k + 2^k = 2 \cdot 2^k = 2^{k+1} \]
\[ \vdash (2k+1)+2 < 2^{k+1} \text{, by substitution.} \]
\[ \vdash 2(k+1)+1 < 2^{k+1} \text{, by substitution.} \]

\[ \vdash \text{By Direct Proof, for all integers } k, \quad k \geq 3 \]
\[ \vdash \text{If } 2k+1 < 2^k \text{, then } 2(k+1)+1 < 2^{k+1} \]
[End of Inductive Step]

\[ \vdash \text{For all integers } n \geq 3, \quad 2n+1 < 2^n, \text{ by Mathematical Induction.} \]

QED