Proposition 6.2.6: For all sets \( A, B, C \),
if \( A \subseteq B \) and \( B \subseteq C^c \), then \( A \cap C = \emptyset \).

Proof: Let \( A, B, C \) be any sets.

Suppose \( A \subseteq B \) and \( B \subseteq C^c \),
[\text{WTS: } A \cap C = \emptyset]

Suppose, by way of contradiction, that \( A \cap C \neq \emptyset \).

\( \therefore \) There exists an element \( x \in A \) such that \( x \in (A \cap C) \).

\( \therefore \) \( x \in A \) and \( x \in C \) by definition of "Intersection".

\( \therefore \) \( x \in C \) by specialization.

\( \therefore \) \( x \in A \) by specialization.

\( \therefore \) Since \( A \subseteq B \), \( x \in B \) by Universal Modus Ponens.

\( \therefore \) Since \( B \subseteq C^c \), \( x \in C^c \) by Universal Modus Ponens.

\( \therefore \) \( x \in C \) by definition of "Complement".

\( \therefore \) \( x \notin C \) and \( x \notin C \) by Conjunction, and this
is a contradiction.

\( \therefore \) \( A \cap C = \emptyset \) by proof-by-contradiction.

QED, By Direct Proof.

The "QED, By Direct Proof" is an abbreviation of the following:

\( \therefore \) For all sets \( A, B, C \), if \( A \subseteq B \) and \( B \subseteq C^c \),
then \( A \cap C = \emptyset \), by Direct Proof.