HW #3 SECTION 3.2 SOLUTIONS
M325K FALL 2017

Sec. 3.2

#2 Negations of "All dogs are loyal"

c. Some dogs are disloyal.
f. There is a dog that is disloyal

#3: a) Find x such that x does not have gills.

b) Find computer c such that c does not have a cpu.

c) Any movie m, m is not over 6 hours long.

d) Any band b, b has not won at least 10 Grammy awards.

#9: The negation of the statement

"For all real numbers x, if x > 3, then x^2 > 9"

is the statement

"Ex real number x such that x > 3 and x^2 \leq 9."
#13. The proposed negation is not a correct negation.

A correct negation is as follows:

There exists an integer $n$ such that $n^2$ is even, but $n$ is not even.

#17. The Negation:

There exists an integer $d$ such that $6/d$ is an integer and $d \neq 3$.

The Negation:

There is an integer which is not odd, but its square is odd.

Also ok: There is an integer whose square is odd, but it is not odd itself.

#34. Original statement:

\[ \forall \text{animals } x, \text{ if } x \text{ is a dog, then } x \text{ has paws and } x \text{ has a tail}. \]

Contrapositive:

\[ \forall \text{animals } x, \text{ if } x \text{ does not have paws or } x \text{ does not have a tail, then } x \text{ is not a dog}. \]
Sec 3.2

#32 ORIGINAL STATEMENT:
"If the square of an integer is odd, then the integer is odd.

This is True.

The Converse:
"If an integer is odd, then its square is odd."

This is True.

The Inverse:
"If the square of an integer is not odd, then the integer is not odd."

This is true.

The Contrapositive:
"If an integer is not odd, then its square is not odd."

This is true.

#412 If one obtains a Master's degree, then they passed a comprehensive exam.
44. The statement is the negation of a conditional:

It is not the case that having a large income is a necessary condition for a person to be happy.

Stated equivalently:

It is not the case that, if one does not have a large income, then one is not a happy person. \( \equiv \) If you are happy, then you have a large income.

Since the negation of a conditional statement is an AND statement \([\neg(p \rightarrow q) \equiv p \land \neg q]\), this can be stated equivalently as:

There exists a person who is happy and does not have a large income.

Informally stated:

Some people without a large income are happy.

This is equivalent to the original statement because

\[ \neg(\forall x, \text{if } P(x), \text{ then } Q(x)) \equiv \exists x \text{ such that } P(x) \land \neg Q(x) \]
§45: The statement is the negation of a conditional:

It is not the case that, if a function is a polynomial function, then it has a root which is a real number.

Because $\neg(\forall x, \text{ if } P(x), \text{ then } Q(x))$

$\equiv$

$\exists x \text{ such that } P(x) \land \neg Q(x)$

This can be stated equivalently as:

There exists a function $f$ such that $f$ is a polynomial function and $f$ does not have a root which is a real number.

Informally: Some polynomial functions do not have a real root.