1. Show that if an \([n, k = Rn]\) binary code is \((\rho, L(n))\) list-decodable, where \(L(n) = 2^{o(n)}\), then \(R \leq 1 - H_2(\rho) + o(1)\).
   
   **Hint:** You may use Lemma 4.8.

2. A \((v, n, \ell)\)-design is a family \(D\) of subsets of \([v]\), such that all \(S \in D\) have size \(n\), and for all \(S \neq T \in D, |S \cap T| \leq \ell\). (This is somewhat different from the classical definition.) Give an explicit \((n^2, n, \ell)\) design with \(n^{\ell+1}\) sets, for infinitely many \(n\) and all nonnegative \(\ell \leq n\).

3. Use concatenation to give, for infinitely many \(n\) and for any \(\epsilon \geq 1/\sqrt{n}\), an explicit \([n, k \geq \epsilon \sqrt{n}, d \geq (1/2 - \epsilon)n]\) binary code. Your construction should be completely explicit, in that it shouldn’t use any greedily-constructed code.

4. For any \(\epsilon > 0\) and positive integer \(k\), construct \(k\) binary random variables \(X_1, \ldots, X_k\) on a probability space of size \(O(k^2/\epsilon^2)\) such that for any non-empty \(S \subseteq [k]\),

\[
| \Pr[\oplus_{i \in S} X_i = 1] - 1/2 | \leq \epsilon.
\]

(As usual, \(\oplus\) denotes the sum mod 2.)

**Hint:** for the upper bound, ensure that your code contains the all 1’s vector.