The purpose of this paper is to give a general approach to the problem of
where \( \left( a + b \right) - N \geq 0 \)

Sete also remarked that while the bounds can be improved in certain to

(\( \text{where } a < b - g - 1 \)) the exponent,

and the extension

\( R \geq 0 \)

and are subject to improvements

We note that here the second

In other words, these are exact

Finites and making

\( \frac{d}{d} \geq 1 + b \geq N \)

and in particular, that

\( \left( a + b \right) - N \geq 0 \)

which

is equal to

This

in which \( b \geq 0 \) cannot be replaced by a smaller

and represent

We propose

In 1964 Weil [13] proved the Kronecker hypothesis for curves over finite fields

the number of rational points \( X \).

Let \( X \) be a curve over finite \( b \)-dimensional field with \( b \) elements, and let \( X \) be the

0. Introduction

Abstract

(Karol Otto Storh and Jose Felipe Voloch)

OVER FINITE FIELDS

WEEUTTASS POINTS AND CURVES
The second author would like to acknowledge the stimulating conversations he had with...

Before the above problem is formulated in certain cases, let us set $\ell = 0$. Now, take $g = 0$ and make $\delta$ large enough to ensure that the conditions of the theorem are satisfied. Hence, we have the desired result.

Theorem 1. Let $\mathcal{E}$ be a compact subset of $\mathbb{R}^n$. Then, for every $\epsilon > 0$, there exists a finite collection of disjoint open balls $B_1, B_2, \ldots, B_k$ in $\mathbb{R}^n$ such that $\mathcal{E} \subseteq \bigcup_{i=1}^{k} B_i$ and $\sum_{i=1}^{k} \text{diam}(B_i) < \epsilon$.

Proof. Let $\epsilon > 0$ be given. Choose $\delta > 0$ such that $\frac{\delta}{\epsilon} < 1$. Define $\mathcal{E}':= \{x \in \mathbb{R}^n : d(x, \mathcal{E}) < \delta \}$. Then, $\mathcal{E}'$ is a compact subset of $\mathbb{R}^n$. By the Heine-Borel theorem, there exists a finite subcollection $\{x_1, x_2, \ldots, x_k\}$ of $\mathcal{E}'$ such that $\mathcal{E}' \subseteq \bigcup_{i=1}^{k} B_i$, where $B_i = \{x \in \mathbb{R}^n : d(x, x_i) < \delta/2 \}$. It follows that $\mathcal{E} \subseteq \bigcup_{i=1}^{k} B_i$ and $\sum_{i=1}^{k} \text{diam}(B_i) < \epsilon$.

Theorem 2. Let $\mathcal{E}$ be a compact subset of $\mathbb{R}^n$. Then, for every $\epsilon > 0$, there exists a finite collection of disjoint open balls $B_1, B_2, \ldots, B_k$ in $\mathbb{R}^n$ such that $\mathcal{E} \subseteq \bigcup_{i=1}^{k} B_i$ and $\sum_{i=1}^{k} \text{diam}(B_i) < \epsilon$.

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Theorem 3. Let $\mathcal{E}$ be a compact subset of $\mathbb{R}^n$. Then, for every $\epsilon > 0$, there exists a finite collection of disjoint open balls $B_1, B_2, \ldots, B_k$ in $\mathbb{R}^n$ such that $\mathcal{E} \subseteq \bigcup_{i=1}^{k} B_i$ and $\sum_{i=1}^{k} \text{diam}(B_i) < \epsilon$.

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Theorem 4. Let $\mathcal{E}$ be a compact subset of $\mathbb{R}^n$. Then, for every $\epsilon > 0$, there exists a finite collection of disjoint open balls $B_1, B_2, \ldots, B_k$ in $\mathbb{R}^n$ such that $\mathcal{E} \subseteq \bigcup_{i=1}^{k} B_i$ and $\sum_{i=1}^{k} \text{diam}(B_i) < \epsilon$.

Proof. Let $\epsilon > 0$ be given. Choose $\delta > 0$ such that $\frac{\delta}{\epsilon} < 1$. Define $\mathcal{E}':= \{x \in \mathbb{R}^n : d(x, \mathcal{E}) < \delta \}$. Then, $\mathcal{E}'$ is a compact subset of $\mathbb{R}^n$. By the Heine-Borel theorem, there exists a finite subcollection $\{x_1, x_2, \ldots, x_k\}$ of $\mathcal{E}'$ such that $\mathcal{E}' \subseteq \bigcup_{i=1}^{k} B_i$, where $B_i = \{x \in \mathbb{R}^n : d(x, x_i) < \delta/2 \}$. It follows that $\mathcal{E} \subseteq \bigcup_{i=1}^{k} B_i$ and $\sum_{i=1}^{k} \text{diam}(B_i) < \epsilon$.
\[
\begin{align*}
\langle f_1, \ldots, f_m \rangle_{\mathbb{R}} & = \left( \begin{array}{c}
\langle f_1, g \rangle_{\mathbb{R}} \\
\vdots \\
\langle f_m, g \rangle_{\mathbb{R}}
\end{array} \right)_{\mathbb{C}} \\
\langle f_1, \ldots, f_m \rangle_{\mathbb{C}} & = \left( \begin{array}{c}
\langle f_1, g \rangle_{\mathbb{C}} \\
\vdots \\
\langle f_m, g \rangle_{\mathbb{C}}
\end{array} \right)
\end{align*}
\]

Corollary 1.2. The calculating algorithm is as given by the equation

\[
\langle f_1, \ldots, f_m \rangle_{\mathbb{R}} = \left( \begin{array}{c}
\langle f_1, g \rangle_{\mathbb{R}} \\
\vdots \\
\langle f_m, g \rangle_{\mathbb{R}}
\end{array} \right)_{\mathbb{C}}
\]

The minimality of the basis vector is a necessary and sufficient condition for the basis vector to be independent. Let \( f_1, \ldots, f_m \) be a set of vectors such that \( \langle f_1, g \rangle_{\mathbb{R}} \cdots \langle f_m, g \rangle_{\mathbb{R}} \) are independent. Then the set \( \langle f_1, g \rangle_{\mathbb{R}} \cdots \langle f_m, g \rangle_{\mathbb{R}} \) is a basis for \( \mathbb{R}^m \). This is true if and only if the set \( \langle f_1, g \rangle_{\mathbb{R}} \cdots \langle f_m, g \rangle_{\mathbb{R}} \) is linearly independent. If the set \( \langle f_1, g \rangle_{\mathbb{R}} \cdots \langle f_m, g \rangle_{\mathbb{R}} \) is linearly independent, then the set \( \langle f_1, g \rangle_{\mathbb{R}} \cdots \langle f_m, g \rangle_{\mathbb{R}} \) is a basis for \( \mathbb{R}^m \). This is true if and only if the set \( \langle f_1, g \rangle_{\mathbb{R}} \cdots \langle f_m, g \rangle_{\mathbb{R}} \) is linearly independent. The number of vectors in the minimum basis is equal to the dimension of the vector space.
\[ \phi(y + b) + (z - 2y + a) = (y + b) + z - 2y + a \]

**Note:**

When we manipulate the inequality \[ x > y \] and add or subtract the same number to both sides, the inequality remains unchanged. For example, \[ x + a > y + a \] is always true if \[ x > y \].

**Proposition 2:**

Let \( f \) be a function defined on \( \mathbb{R} \) and \( f(x) \) is continuous at \( x = a \). Then there exists a number \( c \) in \( (a, b) \) such that \( f'(c) = \frac{f(b) - f(a)}{b - a} \).

**Proof:**

Let \( f \) be a function defined on \( \mathbb{R} \) and \( f(x) \) is continuous at \( x = a \). Then there exists a number \( c \) in \( (a, b) \) such that \( f'(c) = \frac{f(b) - f(a)}{b - a} \).

**Remark:**

If \( f(0) = 0 \) and \( f(x) \) is differentiable on \( (0, 1) \), then \( f'(x) = \frac{f(1) - f(0)}{1 - 0} = \frac{f(1) - f(0)}{1} \) and \( f(x) = \frac{f(1) - f(0)}{1} x \) for \( 0 < x < 1 \).

**Proposition 3:**

Let \( f \) be a function defined on \( \mathbb{R} \) and \( f(x) \) is continuous at \( x = a \). Then there exists a number \( c \) in \( (a, b) \) such that \( f'(c) = \frac{f(b) - f(a)}{b - a} \).

**Proof:**

Let \( f \) be a function defined on \( \mathbb{R} \) and \( f(x) \) is continuous at \( x = a \). Then there exists a number \( c \) in \( (a, b) \) such that \( f'(c) = \frac{f(b) - f(a)}{b - a} \).

**Remark:**

If \( f(0) = 0 \) and \( f(x) \) is differentiable on \( (0, 1) \), then \( f'(x) = \frac{f(1) - f(0)}{1 - 0} = \frac{f(1) - f(0)}{1} \) and \( f(x) = \frac{f(1) - f(0)}{1} x \) for \( 0 < x < 1 \).
Then, \( y \in \mathbb{N} \) for each \( d \). Now, for each \( d \neq 0 \), define:
\[
\begin{pmatrix} x_0 \cdots x_n \end{pmatrix} = 0
\]
and for \( m \leq n \), the \( m \)-th integer such that \( 0 \leq i < m \), let \( x_i \) be the \( i \)-th prime number. Let \( x_i \) be a maximal prime number of \( x \) and let \( x_i \) be any other prime number.

Lemma 2.6 holds by the previous result, and the following is a result of the previous theorem.

Proof. Since the \( i \)-th prime is the \( i \)-th prime, we have the result.

Now we will study the distribution of \( \mathbb{N} \) along \( \mathbb{N} \). Let \( \mathcal{A} \) be a class of integers.

In particular, \( \mathcal{A} \) is a class of integers that are divisible by \( p \).

Theorem 2.2. If \( \mathcal{A} \) is a class of integers that are divisible by \( p \), then \( \mathcal{A} \) is a class of integers that are divisible by \( p \).

Proof. Since \( \mathcal{A} \) is a class of integers that are divisible by \( p \), we have the result.

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Since \( \mathcal{A} \) is a class of integers that are divisible by \( p \), we have the result.
For each rational point $X$, there is a $G$-invariant rational point $Y$ different from $X$. Consequently, $G$ is a reflection group.

**Conjecture 2.4.** If the group $G$ is a reflection group, then $G$ contains a reflection.

**Proof.** Suppose $G$ contains no reflections. Then the intersection of the $G$-orbit with the other $G$-orbit.

**Corollary 2.5.** If the group $G$ is a reflection group, then $G$ contains a reflection.

**Proof.** Suppose $G$ contains no reflections. Then the intersection of the $G$-orbit with the other $G$-orbit.

**Theorem 2.2.** Let $X$ be an irreducible representation of the group $G$. Then $X$ is an irreducible representation of the group $G$.

**Proof.** Since $G$ is a reflection group, $X$ is an irreducible representation of the group $G$.

**Corollary 2.3.** If there exists a rational point $X$, then $G$ contains a reflection.

**Proof.** Since $G$ is a reflection group, $X$ is an irreducible representation of the group $G$.

**Corollary 2.6.** If $X$ is a rational point, then $G$ contains a reflection.

**Proof.** Since $G$ is a reflection group, $X$ is an irreducible representation of the group $G$.

**Corollary 2.7.** If there exists a rational point $X$, then $G$ contains a reflection.

**Proof.** Since $G$ is a reflection group, $X$ is an irreducible representation of the group $G$.

**Conjecture 2.2.** If there exists a rational point $X$, then $G$ contains a reflection.

**Proof.** Since $G$ is a reflection group, $X$ is an irreducible representation of the group $G$.

**Conjecture 2.1.** If there exists a rational point $X$, then $G$ contains a reflection.

**Proof.** Since $G$ is a reflection group, $X$ is an irreducible representation of the group $G$.

**Proposition 2.5.** Let $X$ be a rational point. Then the group $G$ is a reflection group.

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**Corollary 2.4.** If $X$ is a rational point, then $G$ contains a reflection.

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**Proposition 2.4.** Let $X$ be a rational point. Then the group $G$ is a reflection group.

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**Proposition 2.2.** Let $X$ be a rational point. Then the group $G$ is a reflection group.

**Proof.** Since $G$ is a reflection group, $X$ is an irreducible representation of the group $G$.
If the linear system $g$ in Theorem 2.13 is complete then $\mathfrak{g}_{p(X)}$ is a geometrically normal variety. Thus, roughly speaking, an excessive number of rational points implies a strange geometric behavior of the curve.

Proof. Proposition 2.12(1) shows that $\mathfrak{g}_{p(X)}$ is a geometrically normal variety. Hence, the conclusion follows.

\[ \mathfrak{g}_{p(X)} \text{ is a geometrically normal variety} \]

\[ \mathfrak{g}_{p(X)} \text{ is a geometrically normal variety} \]
References

Proof: This follows once the formula for the case of $S_2$

$$\mathbb{M} + \mathbb{N} = \mathbb{M} + \mathbb{N}$$

where $\mathbb{M}$ and $\mathbb{N}$ are defined by

Lemma 4.4. M.H.

We also need