Problem 1. Let $Z$ be a subset of $\mathbb{R}$ with measure zero. Show that the set $A = \{ x^2 : x \in Z \}$ also has measure zero.

Problem 2. Let $f : \mathbb{R}^n \to [0, +\infty]$ a measurable function, denote the measure of set $\Omega \in \mathbb{R}^n$ by $|\Omega|$. Show that

a) $|\{ x \in \mathbb{R}^n : f(x) \geq k \}| \leq \frac{1}{k} \int f$.

b) If $f$ is integrable, then $|\{ x \in \mathbb{R}^n : f(x) = +\infty \}| = 0$.

Problem 3. Let $f$ be of bounded variation on an interval $[a, b]$. If $f = g + h$, where $g$ is absolutely continuous and $h$ is singular, show that

$$\int_a^b \phi \, df = \int_a^b \phi \, f' \, dx + \int_a^b \phi \, dh,$$

for any continuous $\phi$.

Problem 4. Let $\{f_k\}$ be a sequence of non negative measurable function defined on the measurable set $E \in \mathbb{R}^n$.

If $f_k \to f$ and $f_k \leq f$ a.e., show that $\int_E f_k \to \int_E f$.

Problem 5. Let $1 \leq p, q \leq \infty$ with $\frac{1}{p} + \frac{1}{q} = 1$. Show that if $f \in L^p(\mathbb{R}^n)$ and $g \in L^q(\mathbb{R}^n)$, then $f * g$ is bounded and continuous function on $\mathbb{R}^n$. 