1. Let $K$ be a compact connected set in $\mathbb{C}$ that contains the points $\pm i$. Show that there exists a single-valued analytic branch of $(z^2 + 1)^{-1/2}$ on $\mathbb{C} \setminus K$ and determine the possible values of its integral along a closed regular curve in $\mathbb{C} \setminus K$.

2. Let $f$ be analytic and bounded in the upper half plane $H = \{z \in \mathbb{C} : \text{Im } z > 0\}$. Assume that $f(z+1) = f(z)$ for all $z \in H$. Prove that $f(z)$ has a limit as $\text{Im } z \to +\infty$.

3. Let $0 < \alpha < 1$. Show that $\prod_{n=0}^{\infty} \cos(\alpha^n z)$ defines an entire function $f$ of finite order. Determine the order and genus of $f$.

4. Let $U$ and $V$ be two disjoint non-empty open subsets of $\mathbb{C}$. Let $n \mapsto f_n$ be a sequence of analytic functions $f_n : U \to V$. Show that some subsequence converges, locally uniformly, to an analytic function $f$, and that $f$ is one-to-one if each $f_n$ is one-to-one.

(Remark added after the exam. An extra condition is missing: the sequence $n \mapsto f_n(u)$ should be bounded for some $u \in U$.)