The four problems carry equal weights. Full credit requires a complete sequence of arguments that can be understood by a typical student at this level. Partial credit is given for useful approaches and relevant partial results.

1. Determine all analytic functions \( f \) on the strip \( |\text{Im} z| < 2 \) that satisfy \( |f(z)| = |z + 1| \) on the circle \( |z| = 1 \), have simple zeros at \( \pm i/2 \) and no other zeros in the disk \( |z| < 1 \).

2. Describe the set of all functions \( f : \mathbb{C} \setminus \{0\} \to \mathbb{C} \) that are analytic and one-to-one.

3. Determine the partial fraction expansion for the function \( z \mapsto \frac{1}{z \cos \sqrt{z}} \).

4. Let \( D \) be an open subset of \( \mathbb{C} \) and \( R \) a closed proper subset of \( \mathbb{C} \), both non-empty. Let \( n \mapsto f_n \) be a sequence of analytic functions on \( D \) that take values in \( R \). If \( f_n \to f \) pointwise on \( D \), show that \( f \) is analytic on \( D \).