Problem 1.
(a) Let $G$ be a finite group of order $n$. Let $m$ be an integer coprime to $n$. Suppose $g$ and $h$ are elements of $G$ with $g^m = h^m$. Show that $g = h$.
(b) Suppose that $G$ is a finite simple group. Let $p$ be the largest prime dividing $|G|$. Show that $G$ has no proper subgroup $H$ with $[G : H] < p$.

Problem 2. Let $T \in M_n(\mathbb{C})$ be an $n \times n$-matrix such that $T^r$ equals the identity matrix for some integer $r \geq 1$.
Prove that $T$ is conjugate to a diagonal matrix.

Problem 3. Let $A$ be a PID and let $I_1 \supseteq I_2 \supseteq \ldots$ be a strictly decreasing sequence of ideals. Prove that $\bigcap_n I_n = (0)$. (You should prove any non-trivial statements about PIDs that you use in this problem.)