1. Is there an entire function \( f \) that satisfies \( |f(z)| \geq e^{|z|} \) for all values \( z \) large enough? Either provide an example, or prove that none exists.

2. Assume that \( f \) is analytic outside the disk \( \{ z \in \mathbb{C} : |z| \leq 1 \} \) and takes its values inside this disk. Prove that \( |f'(2)| \leq 1/3 \).

3. Suppose \( f \) is analytic on \( \mathbb{D} = \{ z \in \mathbb{C} : |z| < 1 \} \) and satisfies \( |f(z)| \leq M \) for all \( z \in \mathbb{D} \). Assume further that \( f(z) \) vanishes at the points \( \{ z_j \}_{j=1}^{N} \) where \( 1 \leq N \leq \infty \).
   (a) Prove that
   \[
   |f(z)| \leq M \left| \prod_{j=1}^{m} \frac{z - z_j}{1 - \bar{z_j} z} \right| \quad \forall z \in \mathbb{D},
   \]
   for any \( 1 \leq m \leq N \) (or if \( N = \infty \), then \( 1 \leq m < N \)).
   (b) If \( N = \infty \) and \( f \neq 0 \), then show that
   \[
   \sum_{j=1}^{\infty} (1 - |z_j|) < \infty.
   \]

4. Prove that
   \[
   \frac{\pi^2}{\sin^2(\pi z)} = \sum_{n=-\infty}^{\infty} \frac{1}{(z - n)^2}.
   \]