Extension of Range of Validity of Pernici's 1/n Conjecture

Paul Federbush
Department of Mathematics
University of Michigan
Ann Arbor, MI 48109-1043
pfed@umich.edu

April 29, 2022

Abstract

We understand Pernici's 1/n conjecture to be eqs. (16) and (17) of his paper "Expansion for the Number of Matchings on Regular Graphs and Monomer-Dimer Entropy", [1]. We have proven a generalized form of eq.(16) in [2]. In this note we extend the range of validity of (17) from $j \leq 100$, $r \leq 10$, shown by computer computation in [1], with the range $j \leq 29$, all r, also shown by computer computation. Naturally we are using the notation of [1].

We first recall eqs. (16) and (17) from [1].

$$[j^k n^{-h}] \ln\left(1 + H_j\right) = [j^k n^{-h}] \ln\left(1 + \sum_{s=1}^{j-1} \frac{a_s(r,j)}{n^s}\right) = 0, \qquad k \geq h+2$$

$$[j^{h+1} n^{-h}] \ln\left(1 + H_j\right) = [j^{h+1} n^{-h}] \ln\left(1 + \sum_{s=1}^{j-1} \frac{a_s(r,j)}{n^s}\right) = \frac{1}{(h+1)h}\left(\frac{1}{r^h} - 2\right)$$

We are working with j-matchings on an r-regular bipartite lattice with 2n vertices. To understand the remaining notation in these equations one need only read eq.(3) and the text near this equation, and eqs.(10) - (15). all in [1] of course. After eq.(20) Pernici states he has computer checked these equations for $j \leq 100$, $r \leq 10$. We have proven a generalized form of eq.(16) in [2]. For $h \leq 23$ we directly computer checked the two sides of the second equation are equal functions of r. The two sides are polynomials in r of degree h. By substituting r=1,2,......,h+1 in the two sides and computer checking the two sides are numerically equal, one verified the equation for $h \leq 29$. One is working with rational functions of all the variables, integer arithmetic on the computer. I worked with Maple on my desktop computer with 8 M memory. My limiting values were determined by running out of memory, clearly better equipment and technical improvements in the program should be expected to extend the limits. The time and memory use grew surprisingly slowly with h, (at least I was surprised). Where are the good graduate students who should be doing these calculations? Pernici's paper is fine mathematical physics.

References

[1] M. Pernici, "$\frac{1}{n}$ Expansion for the Number of Matchings on Regular Graphs and Monomer-Dimer Entropy", J. Stat. Phys 168, 666 (2017)

[2] P. Federbush, "A PROOF of Weak Graph Positivity for a Large Range of the Parameters", arXiv: 2012.10927.