Multiscale Analysis of Vibrations of Streamers

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Joint work with
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Outline of presentation

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- The AP stretch - streamer system.

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- Determining streamer impedance.

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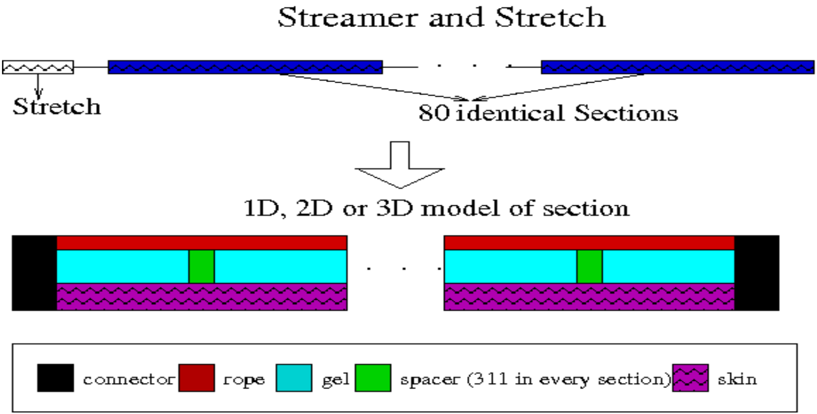
- Determining streamer impedance.
- Multiscale analysis of streamers.
- Dual-Mixed formulation for elasticity.
- Additional background info:
  - Coupled elasticity-acoustics problem,
  - Exact sequence. Projection-Based Interpolation.
  - Automatic hp-Adaptivity and coupled multiphysics problems.

Surveying the Ocean Floor with Acoustical Streamers

An array of 8 streamers, 6km long, pulled by a tugboat

The AP Stretch - Streamer System

Geometry of the AP Stretch – Streamer System



Material Data for the Streamer

component	E(GPa)	ρ(kg.m^{-3})	ν	Length(m)	height(m)
Connector	200	8000	0.29	0.0975	0.0307
Spacer	1.8	1200	0.30	0.075	0.021843
Rope	41.0	1400	0.30	75	0.005657
Skin	0.02	1200	0.45	75	0.0032
Gel	$\text{E}_{\text{gel}}(\omega, T)$	1040	0.45	0.165	0.021843

$$E_{gel}(\omega, T) = E_r(\omega, T) + iE_r(\omega, T).$$

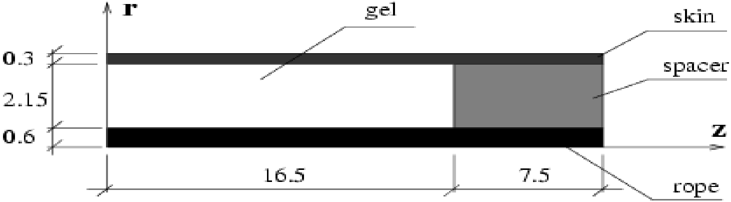
At $T = 10°C$, we have:

$$E_r(\omega, T) = 2.9 \times 10^{0.4125 \log_{10}(\omega) + 3.0871} \, [Pa]$$

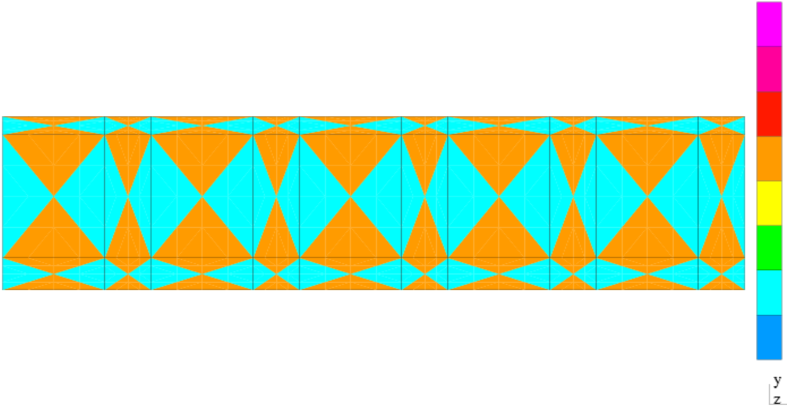
$$E_i(\omega, T) = 2.9 \times 10^{0.3977 \log_{10}(\omega) + 2.9707} \, [Pa]$$

2D Model: Axisymmetric Elasticity

$$\begin{cases} \text{Find } \boldsymbol{u} \text{ such that} & \\ -\rho\omega^2 u_i - (E_{ijkl}u_{k,l})_{,j} = 0 & \text{in } \Omega \\ u_i = u_{D,i} & \text{on } \Gamma_D \\ E_{ijkl}u_{k,l}n_j + i\omega\rho_f c^2 = 0 & \text{on } \Gamma_C \end{cases}$$

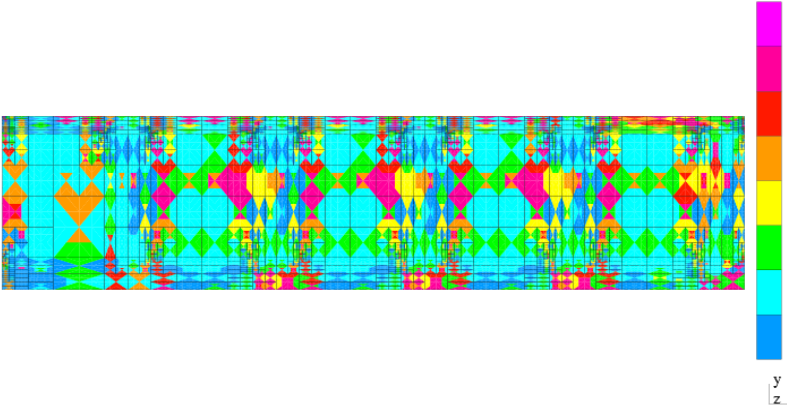


2D Model: Axisymmetric Elasticity



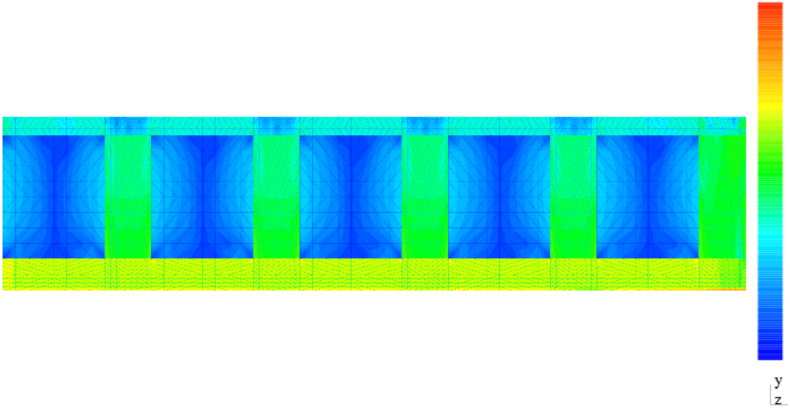
Initial mesh

2D Model: Axisymmetric Elasticity



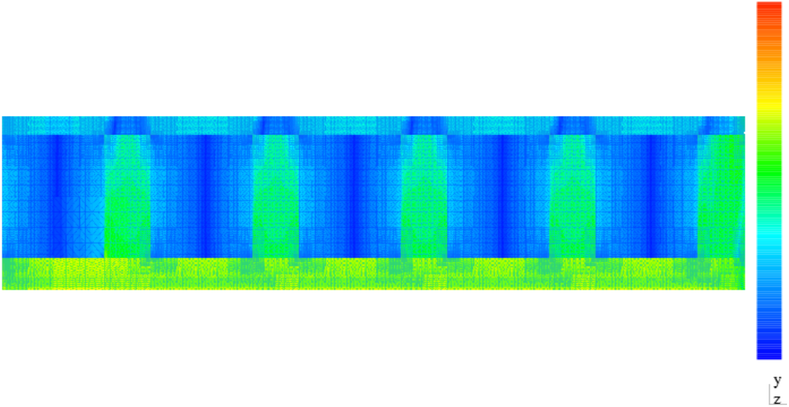
Optimal mesh corresponding to 2% error in energy

2D Model: Axisymmetric Elasticity



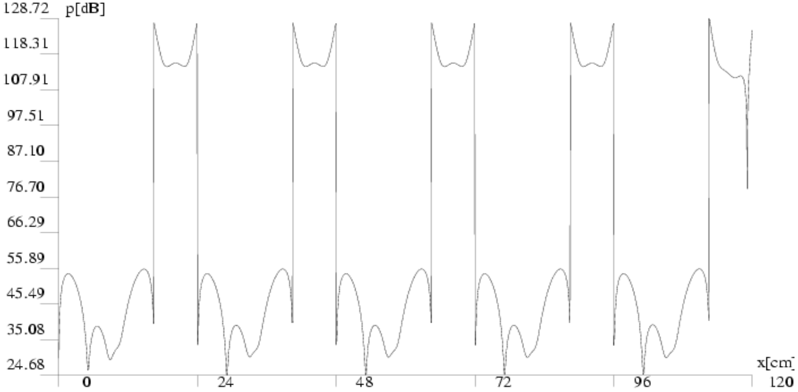
Pressure contours for the initial mesh

2D Model: Axisymmetric Elasticity



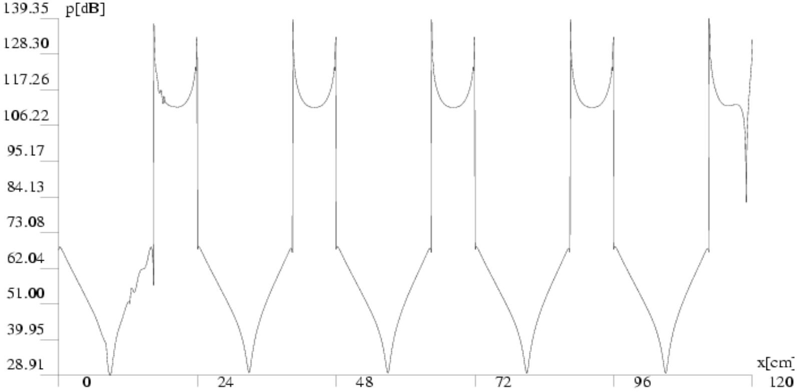
Pressure contours for the optimal mesh

2D Model: Axisymmetric Elasticity



Pressure profile for the initial mesh

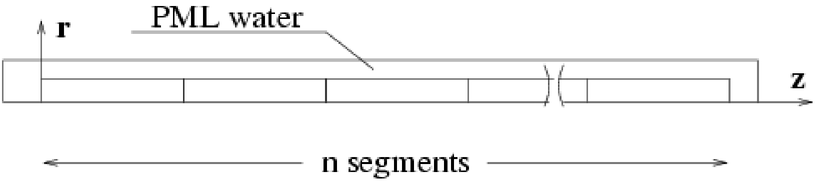
2D Model: Axisymmetric Elasticity



Pressure profile for the optimal mesh

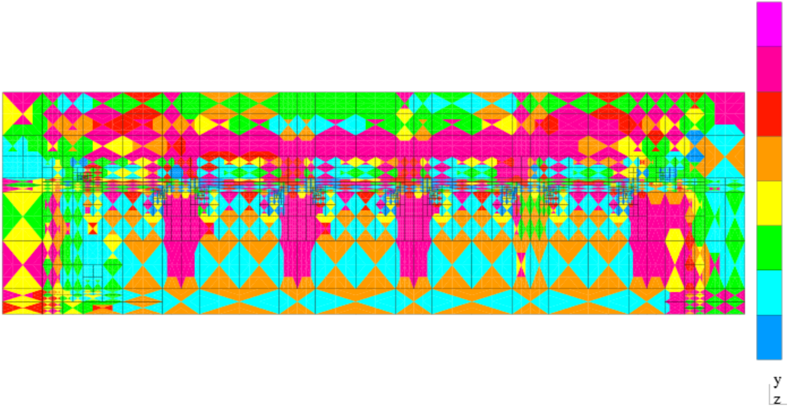
2D Model: Coupled Elasticity/Acoustics

$$
\begin{cases}
\text{Find } \boldsymbol{u}\text{,p such that} & \\
-\rho\omega^2 u_i - (E_{ijkl}u_{k,l})_{,j} = 0 & \text{in } \Omega_s \\
-p_{,ii} - \left(\frac{\omega}{c}\right)^2 p = 0 & \text{in } \Omega_f \\
u_i = u_{D,i}, p_{,i}n_i = \rho_f\omega^2 u_{D,i}n_i & \text{on } \Gamma_D \\
E_{ijkl}u_{k,l}n_j = -pn_i, p_{,i}n_i = \rho_f\omega^2 u_i n_i & \text{on } \Gamma_I \\
\;+ \text{ Sommerfeld radiation condition at } \infty &
\end{cases}
$$



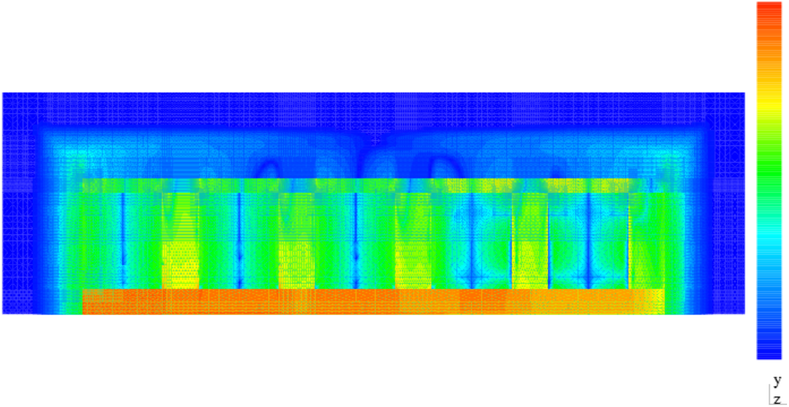
The structure is axisymmetric.

2D Model: Coupled Elasticity/Acoustics



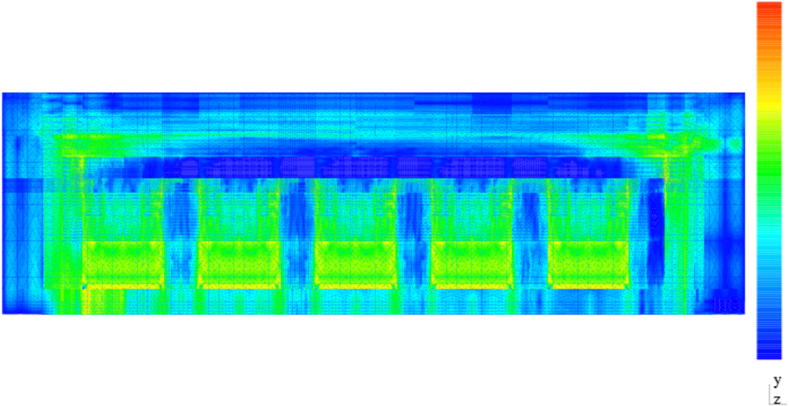
Optimal hp mesh corresponding to 3.6 percent error

2D Model: Coupled Elasticity/Acoustics



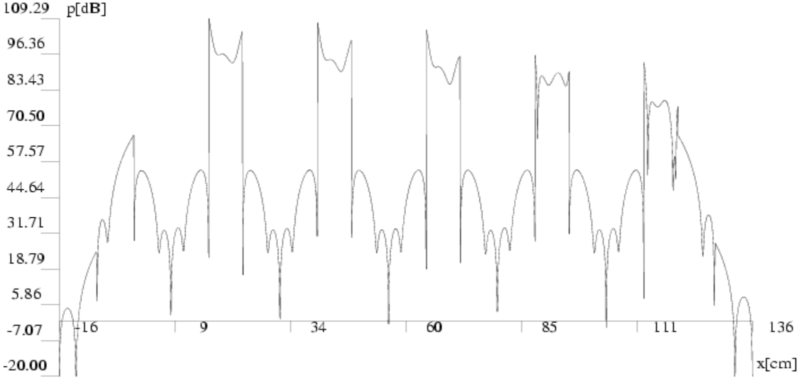
Pressure distribution on the optimal mesh. Range: -20 - 133.6 dB

2D Model: Coupled Elasticity/Acoustics



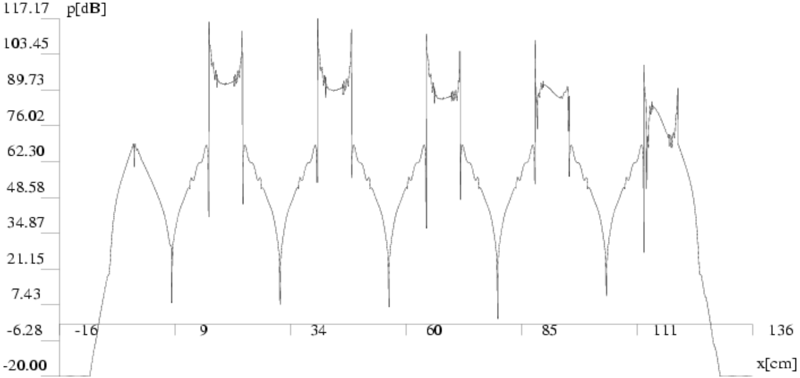
Difference between coarse and fine grid pressures in dB

2D Model: Coupled Elasticity/Acoustics



Pressure profile for the initial mesh

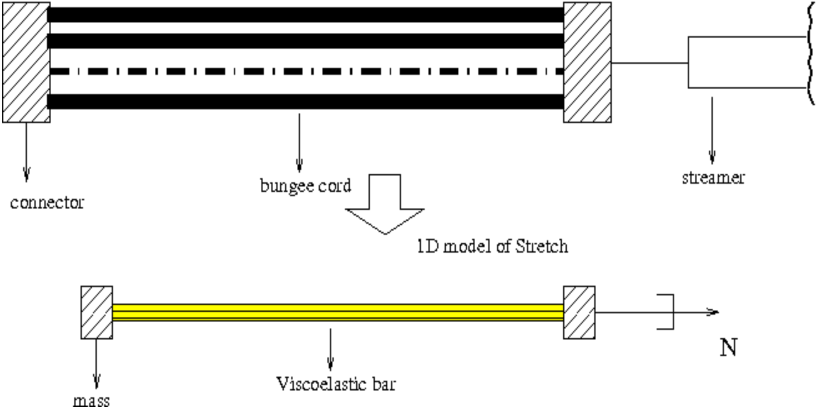
2D Model: Coupled Elasticity/Acoustics



Pressure profile for the optimal mesh

Determining Streamer Impedance

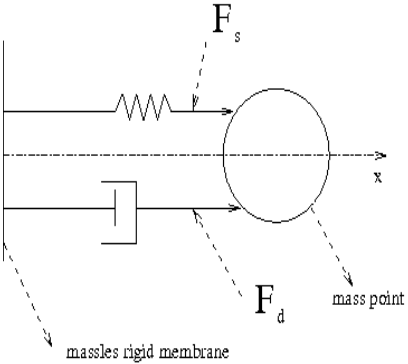
Streamer Impedance



Define streamer impedance as $\beta = \frac{N}{u}$ where N is the force across the stretch/streamer interface , and u is the displacement of the interface. $N = \beta u$ provides a Cauchy B.C. for a 1D Stretch model.

Task: Compute impedance β as function of frequency ω and water temperature.

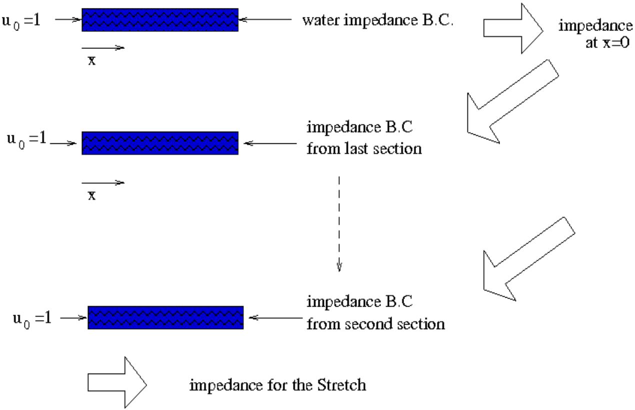
Two D.O.F. Model of the Streamer



$$\beta = \frac{F}{u_0} = \frac{\omega^2(k + i\omega c)}{-\omega^2 + i\omega c + k}$$

Impedance is singular at resonant frequencies

Iterative Procedure to Compute the Streamer Impedance



Computation of Impedance for a Single Section

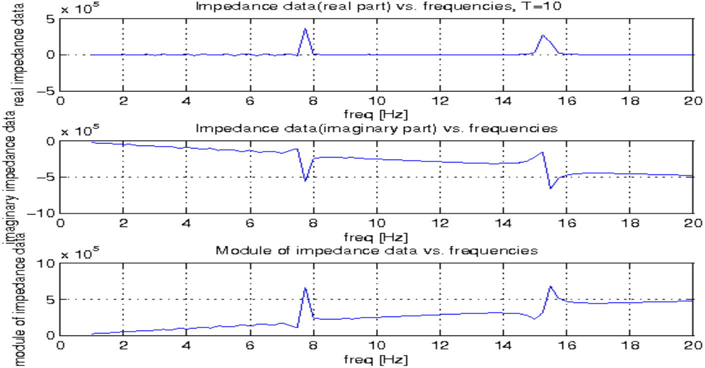
Integrate by parts:

$$\begin{aligned} b(\boldsymbol{u},\boldsymbol{v}) &= \int_\Omega (\sigma_{ij} v_{i,j} - \rho\omega^2 u_i v_i)\, dx + \int_{\Gamma_C} \beta_{ij} u_j v_i dS \\ &= \int_\Omega (\sigma_{ij,j} - \rho\omega^2 u_i)\, v_i\, dx + \int_{\Gamma_D} t_i v_i\, dS + \int_{\Gamma_C} \beta_{ij} u_j v_i dS \end{aligned}$$

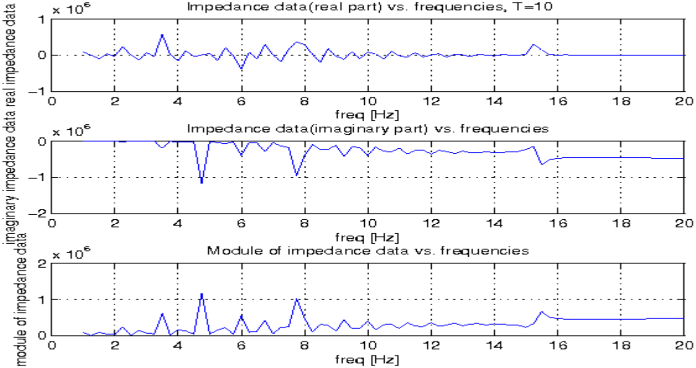
Choose a special test function $\boldsymbol{v} = (v_z, v_r)$ where $v_z = 1, v_r = 0$ on Γ_D and $\boldsymbol{v} = 0$ on the rest of the boundary. It follows that:

$$\int_{\Gamma_D} t_z dS = b(\boldsymbol{u},\boldsymbol{v}) \approx b(\boldsymbol{u}_{hp},\boldsymbol{v})$$

Numerical Results: 2D Elasticity Model

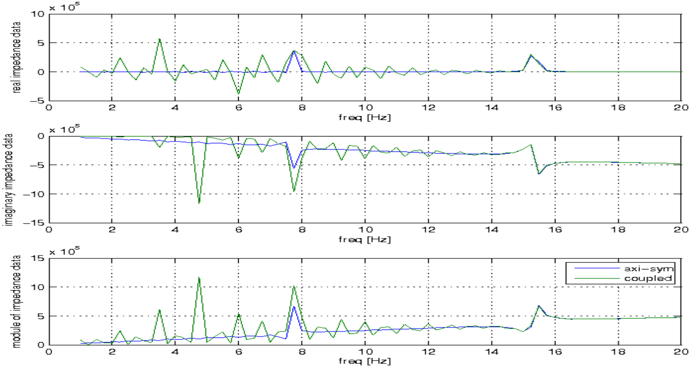


Streamer impedance as a function of frequency. $T = 10^o$ C



Streamer impedance as a function of frequency. $T = 10^o$ C

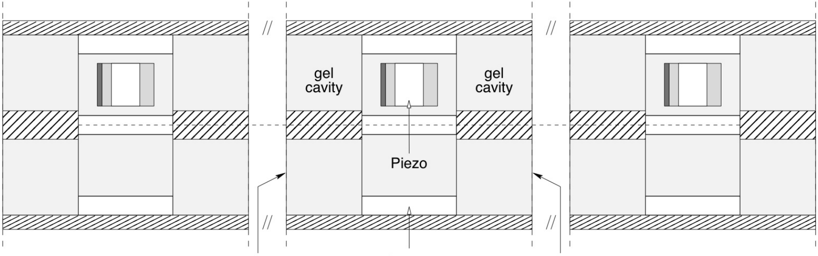
Comparison of Elastic and Coupled Models



Streamer impedance as a function of frequency. T $= 10^o$ C

Local Analysis of Hydrophones
A Multiscale Approach

3D Model: Local Analysis of Hydrophones

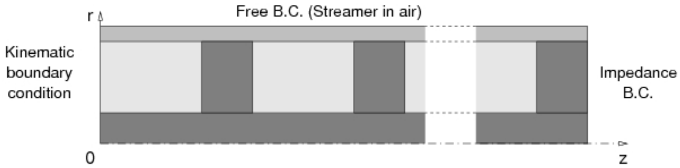


Task: Analyze pressure distribution around the microphones

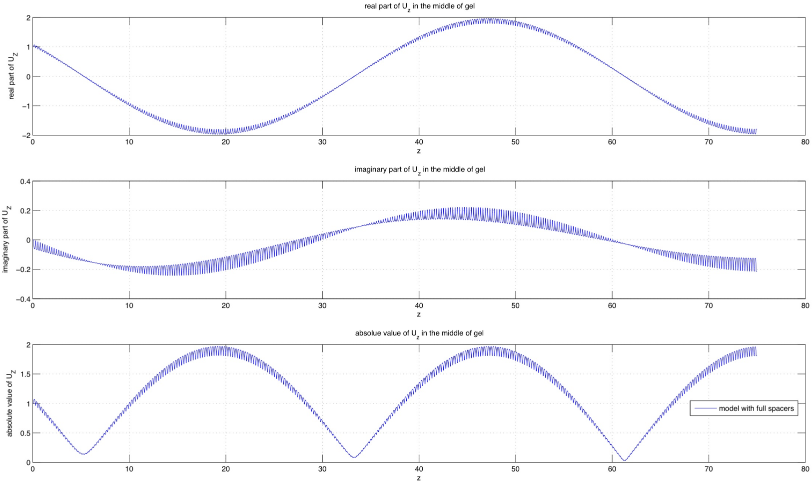
Issue: How to define appropriate boundary conditions ? (Periodic ?)

Axi-symmetric Streamer Section

In order to determine the BC on one periodic cell, we studied the response on a 75m long streamer section. We used the axisymmetric model for detailed studies, but results were confirmed on the 3D simplified model.
In particular we measured the displacement along the streamer and accross the two-end sections of one cell in the streamer.
Results are shown in following slides.

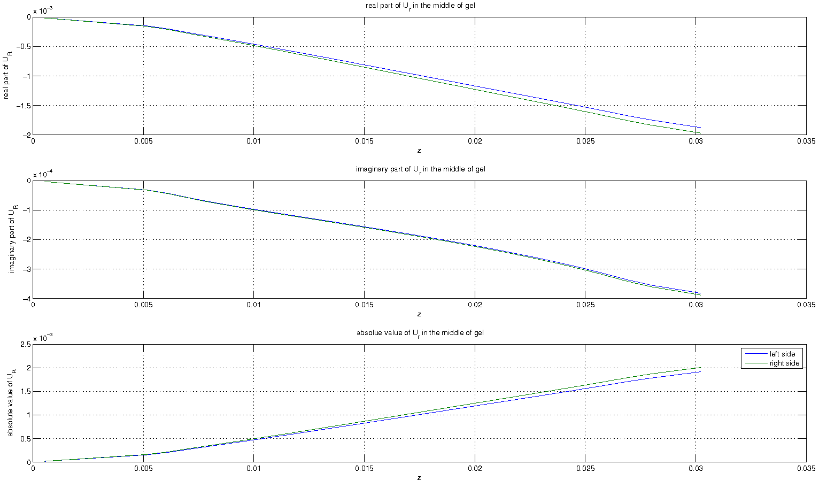


Study of 75m Streamer Section



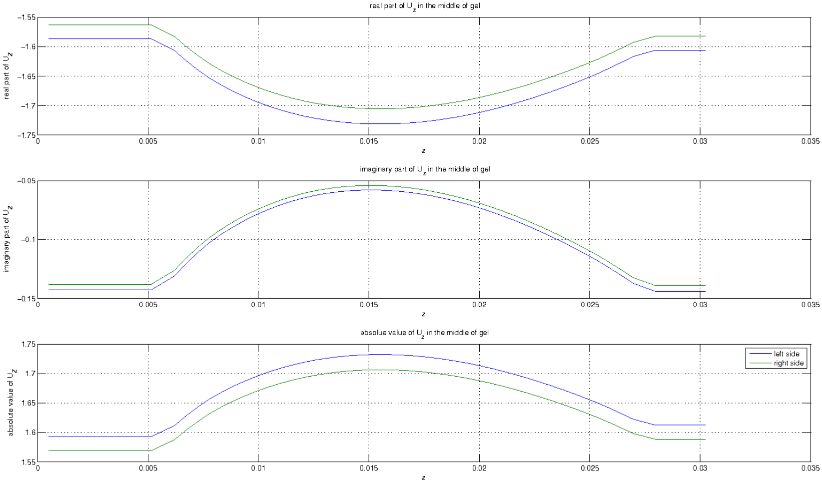
Displacement u_z (real part, imaginary part, amplitude) along streamer.

Study of 75m Streamer Section



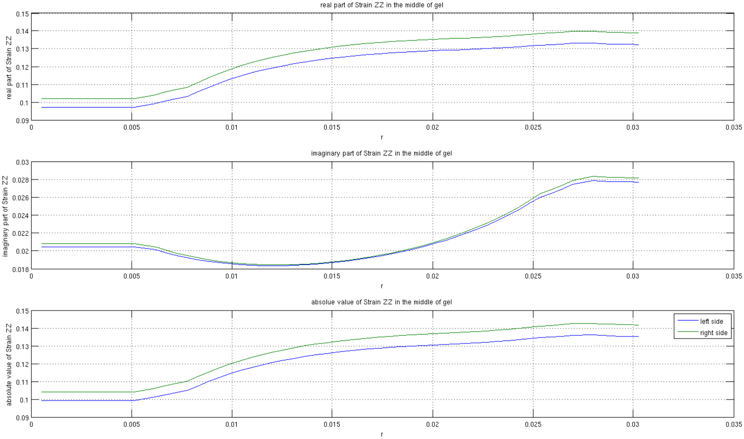
Displacement u_r (real part, imaginary part, amplitude) across mid-section in the gel cavities before and after one spacer.

Study of 75m Streamer Section



Displacement u_z (real part, imaginary part, amplitude) across mid-section in the gel cavities before and after one spacer.

Study of 75m Streamer Section



Strain ϵ_{zz} (real part, imaginary part, amplitude)
across mid-section in the gel cavities before and after one spacer.

Observations from Numerical Study

1. This is clearly a two-scale problem:

   $$u(x) = U(x) + u_s(x)$$

2. The small scales u_s are proportional to the large scales and not to their derivatives.

3. $$\begin{aligned} u_r^R &= u_r^L \\ u_z^R &= u_z^L + C \\ \mathrm{Re}(\epsilon_{zz})^R &= \mathrm{Re}(\epsilon_{zz})^L + C \\ \mathrm{Im}(\epsilon_{zz})^R &\approx \mathrm{Im}(\epsilon_{zz})^L + C \end{aligned}$$

4. The displacement of the small scales in one cell is symmetric with respect to the cross-section that passes through the middle of the spacer (see zoom below).

Some Mathematics

Hypothesis:

The small scale u_s satisfies periodic boundary conditions.

Case 1: Periodic kinematic BC: $u_s^R = u_s^L + \delta u$, $\epsilon_{s,zz}^R = \epsilon_{s,zz}^L$
Case 2: Periodic strain BC: $u_s^R = u_s^L$, $\epsilon_{s,zz}^R = \epsilon_{s,zz}^L + \delta\epsilon$

For a symmetric geometry of the spacer, it can be shown that Case 1 necessarily yields an antisymmetric solution and Case 2 a symmetric solution. So we choose Case 2.

Global/local ansatz: u_s and u are now approximated by $\bar{u}_s$ and $\bar{u}$ such that

$$\bar{u}_s \approx CU, \text{ with } U = (0, 0, U_z) \text{ and } C = \text{constant}$$

so

$$\bar{u} = U + \bar{u}_s \approx u \quad \text{and} \quad \delta\epsilon \propto U$$

Solution of the Local Problem

- The small scale (solution of the local problem) is driven by the large scale represented by complex constant C in the BC. It changes with the location of the spacer/microphone.

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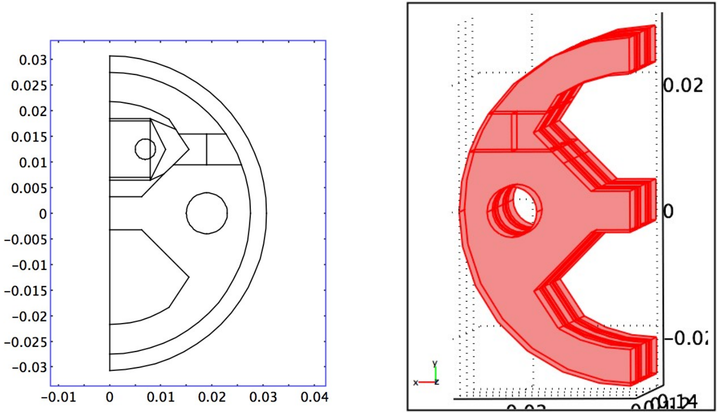
- Quantities like ratio of pressures at different locations or phase difference are independent of constant C and can be evaluated w/o its knowledge.

Solution of the Local Problem

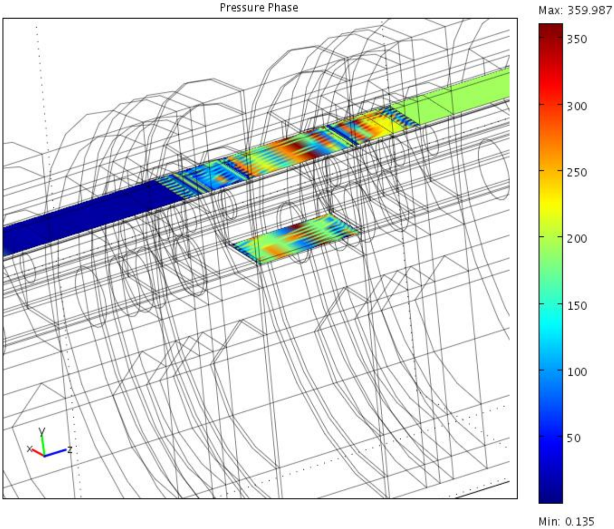
- The small scale (solution of the local problem) is driven by the large scale represented by complex constant C in the BC. It changes with the location of the spacer/microphone.
- Quantities like ratio of pressures at different locations or phase difference are independent of constant C and can be evaluated w/o its knowledge.
- Pressure (derivatives) depends mainly upon the small scale.

Comsol Problem Setting



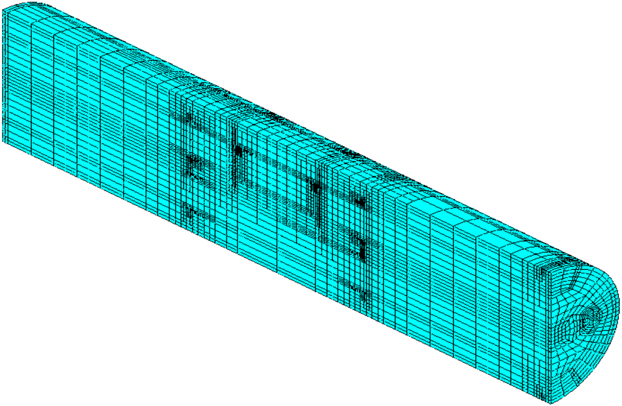
Geometry of Hydrophone

Solution with Comsol



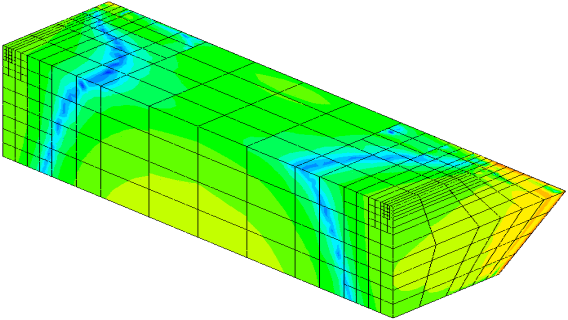
Pressure phase on the slice which passes on top of the hydrophone.

Verification with $hp3d$



Goal-driven h-Adaptivity: 8th fine mesh, 31k elements, 730k dofs
Goal: Average pressure over the microfone.

Verification with $hp3d$



Pressure Over the Microfone: min/max=67/137 [dB]

Dual-Mixed Formulation for Elasticity

Dual-Mixed Formulation for Elasticity

$$\sigma_{ij} = E_{ijkl}\epsilon_{kl} = E_{ijkl}(u_{k,l} - \omega_{kl}) \Longrightarrow$$
$$C_{klij}\sigma_{ij} = u_{k,l} - \omega_{kl}$$
$$\sigma_{ij,j} + \rho\omega^2 u_i = f_i$$

$$\left\{ \begin{array}{lll} \sigma_{ij}n_j = g_i \text{ on } \Gamma_t & & \\ \int_\Omega C_{klij}\sigma_{ij}\tau_{kl} + \int_\Omega u_k\tau_{kl,l} + \int_\Omega u_k\omega_{kl}\tau_{kl} & = \int_{\Gamma_u} \bar{u}_k\tau_{kl}n_l & \forall\tau_{kl} \; : \\ & & \tau_{kl}n_l = 0 \text{ on } \Gamma_t \\ \int_\Omega \sigma_{ij,j}v_i + \rho\omega^2\int_\Omega u_i v_i & = \int_\Omega f_i v_i & \forall v_i \\ \int_\Omega \sigma_{kl}q_{kl} & = 0 & \forall q_{kl} = -q_{lk} \end{array} \right.$$

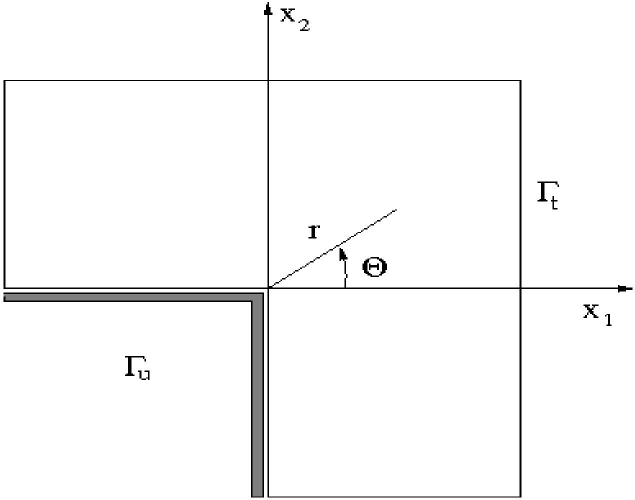
Elasticity Complex (Arnold at al. "decoded")

$$\begin{array}{ccccccccccc}
\Phi(\boldsymbol{W}) & \hookrightarrow & \Lambda^0(\boldsymbol{W}) & \xrightarrow{\mathcal{A}_0} & \Lambda^1(\boldsymbol{W}) & \xrightarrow{\mathcal{A}_1} & \Lambda^2(\boldsymbol{W}) & \xrightarrow{\mathcal{A}_2} & \Lambda^3(\boldsymbol{W}) & \longrightarrow & 0 \\
\downarrow id & & \downarrow id & & \downarrow \pi^1 & & \downarrow \pi^2 & & \downarrow id & & \\
\Phi(\boldsymbol{W}) & \hookrightarrow & \Lambda^0(\boldsymbol{W}) & \xrightarrow{\mathcal{A}_0} & \Gamma^1 & \xrightarrow{\mathcal{A}_1} & \Gamma^2 & \xrightarrow{\mathcal{A}_2} & \Lambda^3(\boldsymbol{W}) & \longrightarrow & 0
\end{array}$$

$$\boldsymbol{W} = \boldsymbol{K} \times \boldsymbol{V}, \quad \boldsymbol{K} = \{\omega^{ij} = \epsilon^{ijk}\Psi^k\}, \quad \boldsymbol{V} = \{\phi^i\}$$

$$\begin{aligned}
\Lambda^0(\boldsymbol{W}) &= \{(\Phi^m, \phi^i)\} \\
\Lambda^1(\boldsymbol{W}) &= \{(E^m_k, e^i_k)\} \quad \Gamma^1 = \{(E^m_k, e^i_k) \;:\; \epsilon_{nlk}E^m_{k,l} - (e^n_m - e^k_k\delta^n_m) = 0\} \\
\Lambda^2(\boldsymbol{W}) &= \{(V^m_n, v^i_m)\} \quad \Gamma^2 = \{(0, v^i_m)\} \\
\Lambda^3(\boldsymbol{W}) &= \{(\Psi^m, \psi^i)\} \\
\mathcal{A}_0(\Psi^m, \phi^i) &= (E^m_k, e^i_k) \qquad = (\Phi^m_{,k} + \epsilon^{mk\alpha}\phi^\alpha, \phi^i_{,k}) \\
\mathcal{A}_1(E^m_k, e^i_k) &= (V^m_n, v^i_m) \qquad = (\epsilon_{nlk}E^m_{k,l} - (e^n_m - e^k_k\delta^n_m), \epsilon_{mlk}e^i_{k,l}) \\
\mathcal{A}_2(V^m_n, v^i_m) &= (\Psi^m, \psi^i) \qquad = (V^m_{n,n} - \tfrac{1}{2}\epsilon^{min}v^i_n, v^i_{m,m})
\end{aligned}$$

p-convergence Test

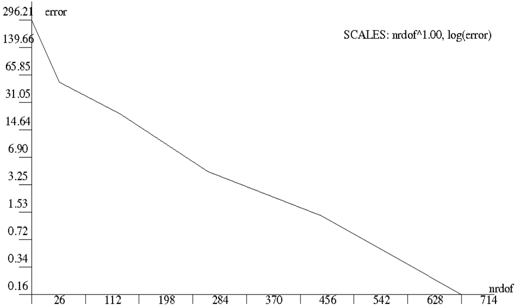


Geometry and boundary conditions.

Regular Solution

Manufactured solution:

$$u_1 = \cos(x + 2y) \quad u_2 = \sin(3x + y)$$

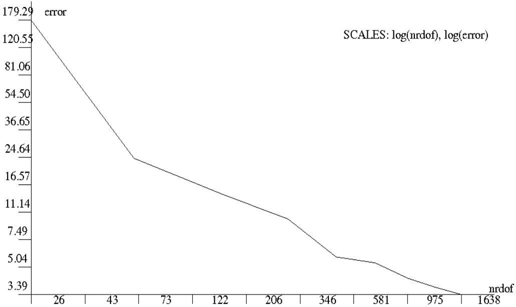


Convergence rates (ln error vs. number of d.o.f.).

Singular Solution

Manufactured solution:

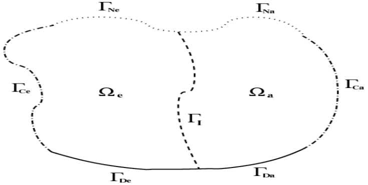
$$u_1 = u_2 = r^\alpha \sin(\alpha(\theta + \frac{\pi}{2})), \quad \alpha = 1.34$$



Convergence rates (ln error vs. ln d.o.f.).

Additional Background Info

Coupled Acoustics/Elasticity



acoustics in Ω_a :

$$\begin{cases} c^{-2} i\omega p + \rho_f w_{i,i} &= 0 \\ \rho_f i\omega w_i + p_{,i} &= 0 \end{cases}$$

elasticity in Ω_e :

$$\begin{cases} -\rho_s \omega^2 u_i - \sigma_{ij,j} = 0 \\ \sigma_{ij} = \mu(u_{i,j} + u_{j,i}) + \lambda u_{k,k}\delta_{ij} \end{cases}$$

$i\omega u_i n_i = w_i n_i \quad \sigma_{ij} n_j = -p n_i$ on interface Γ_I

$+$ standard boundary conditions on the boundary

Weak Coupling

Step 1: Formulate conservation of mass (acoustics) and balance of momentum (elasticity) in a weak form:

$$\begin{aligned} -\int_{\Omega_a} \left(\frac{\omega}{c}\right)^2 pq + i\omega\rho_f v_i q_{,i} - \int_{\Gamma_I} i\omega\rho_f v_i n_i q &= \text{B.T.} \quad \forall q \\ \int_{\Omega_e} -\omega^2 \rho u_i v_i + \sigma_{ij} v_{i,j} - \int_{\Gamma_I} \sigma_{ij} n_j v_i &= \text{B.T.} \quad \forall v_i \end{aligned}$$

Weak Coupling

Step 1: Formulate conservation of mass (acoustics) and balance of momentum (elasticity) in a weak form:

$$\begin{aligned} -\int_{\Omega_a} \left(\frac{\omega}{c}\right)^2 pq + i\omega\rho_f v_i q_{,i} - \int_{\Gamma_I} i\omega\rho_f v_i n_i q &= \text{B.T.} \quad \forall q \\ \int_{\Omega_e} -\omega^2 \rho u_i v_i + \sigma_{ij} v_{i,j} - \int_{\Gamma_I} \sigma_{ij} n_j v_i &= \text{B.T.} \quad \forall v_i \end{aligned}$$

Step 2: Use the remaining equations in the strong form to eliminate fluid velocity and elastic stresses:

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\end{aligned}
$$

Step 3: Use the interface conditions to couple the two variational formulations:

$$
\begin{aligned}
-\int_{\Omega_a} \left(\frac{\omega}{c}\right)^2 pq + p_{,i} q_{,i} + \omega^2 \int_{\Gamma_I} \rho_f u_i n_i q &= \text{B.T.} \quad \forall q \\
\int_{\Omega_e} -\omega^2 \rho u_i v_i + \mu(u_{i,j} + u_{j,i}) v_{i,j} + \lambda u_{k,k} v_{k,k} + \int_{\Gamma_I} p v_i n_i &= \text{B.T.} \quad \forall v_i
\end{aligned}
$$

Abstract Variational Formulation

$$\left\{ \begin{array}{lll} \boldsymbol{u} \in \tilde{\boldsymbol{u}}_D + \boldsymbol{V}_e, \ p \in \tilde{p} + V_a & & \\ b_{ee}(\boldsymbol{u}, \boldsymbol{v}) + b_{ae}(p, \boldsymbol{v}) & = l_e(\boldsymbol{v}) & \forall \boldsymbol{v} \in \boldsymbol{V}_e \\ b_{ea}(\boldsymbol{u}, q) + b_{aa}(p, q) & = l_a(q) & \forall q \in V_a \end{array} \right.$$ where

$$\begin{aligned} \boldsymbol{V}_e &= \{\boldsymbol{v} \in \boldsymbol{H}^1(\Omega_e) \ : \ \boldsymbol{v} = \boldsymbol{0} \text{ on } \Gamma_{De}\} \\ V_a &= \{q \in H^1(\Omega_a) \ : \ q = 0 \text{ on } \Gamma_{Da}\} \\ b_{ee}(\boldsymbol{u}, \boldsymbol{v}) &= \int_{\Omega_e} (E_{ijkl} u_{k,l} v_{i,j} - \rho_s \omega^2 u_i v_i) \, d\boldsymbol{x} + \int_{\Gamma_{Ce}} \beta_{ij} u_i v_j \, dS \\ b_{ae}(p, \boldsymbol{v}) &= \int_{\Gamma_I} p v_n \, dS \\ b_{ea}(\boldsymbol{u}, q) &= -\int_{\Gamma_I} u_n q \, dS \\ b_{aa}(p, q) &= \frac{1}{\omega^2 \rho_f} \int_{\Omega_a} (\boldsymbol{\nabla} p \boldsymbol{\nabla} q - k^2 pq) \, d\boldsymbol{x} \\ l_e(\boldsymbol{v}) &= \int_{\Omega_e} f_i v_i \, d\boldsymbol{x} + \int_{\Gamma_{Ne} \cup \Gamma_{Ce}} g_i v_i \, dS \\ l_a(q) &= \int_{\Omega_a} f q \, d\boldsymbol{x} + \int_{\Gamma_{Na} \cup \Gamma_{Ca}} g v \, dS \end{aligned}$$

For scattering problems:

$$l_e(\boldsymbol{v}) = -\int_{\Gamma_I} p^{inc} v_n \, dS, \quad l_a(q) = -\frac{1}{\omega^2 \rho_f} \int_{\Gamma_I} \frac{\partial p^{inc}}{\partial n} q \, dS$$

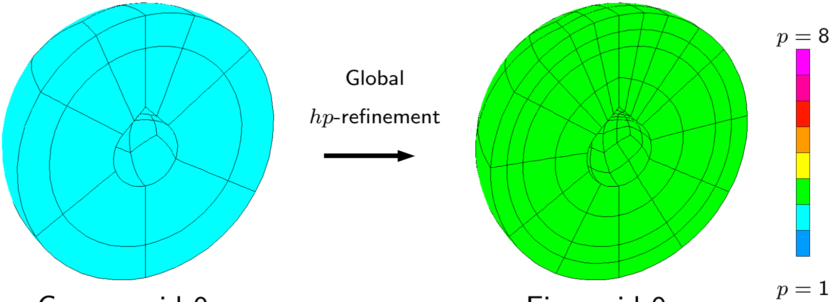
de Rham Diagram

$$
\begin{array}{ccccccccccc}
\mathbb{R} & \rightarrow & H^1 & \xrightarrow{\nabla} & \boldsymbol{H}(\text{curl}) & \xrightarrow{\nabla\times} & \boldsymbol{H}(\text{div}) & \xrightarrow{\nabla\circ} & L^2 & \longrightarrow & \mathbf{0} \\
\big\downarrow id & & \big\downarrow \Pi^{\text{grad}} & & \big\downarrow \Pi^{\text{curl}} & & \big\downarrow \Pi^{\text{div}} & & \big\downarrow P & & \\
\mathbb{R} & \longrightarrow & W_{hp} & \xrightarrow{\nabla} & \mathbf{Q}_{hp} & \xrightarrow{\nabla\times} & \mathbf{V}_{hp} & \xrightarrow{\nabla\circ} & Y_{hp} & \longrightarrow & \mathbf{0}
\end{array}
$$

where the *Projection-Based Interpolation Operators* $\Pi^{\text{grad}}, \Pi^{\text{curl}}, \Pi^{\text{div}}$, and L^2-projection P make the diagram commute.

A Two-Grid Paradigm

Our approach is to determine an optimal refinement strategy for a given *coarse grid* by examining the solution on a corresponding *fine grid* obtained by a global hp-refinement.



Coarse grid 0 Fine grid 0

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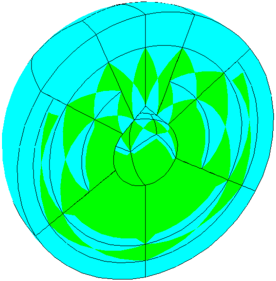
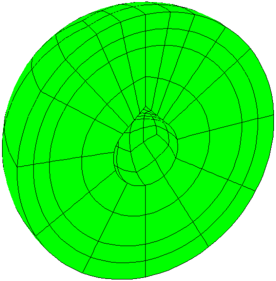
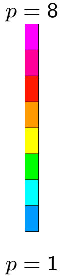


Coarse grid 1

Error estimate and optimal hp-refinement



Fine grid 0



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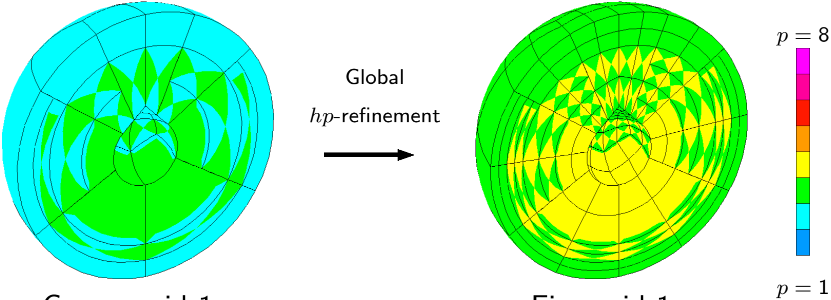

Coarse grid 1 Fine grid 1

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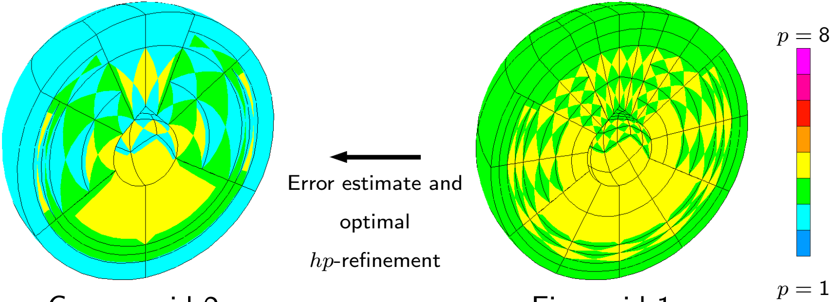

Coarse grid 2

Fine grid 1

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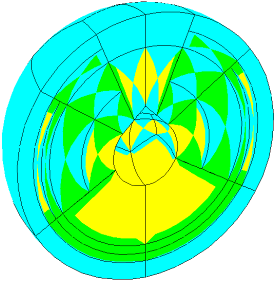
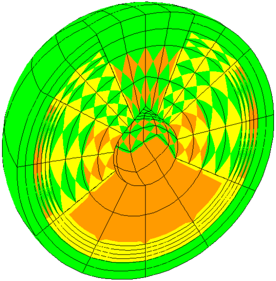


Coarse grid 2

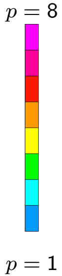
Global

hp-refinement

→



Fine grid 2



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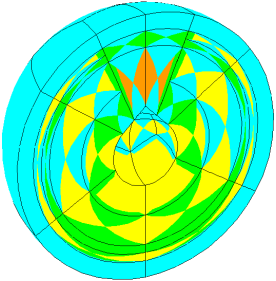
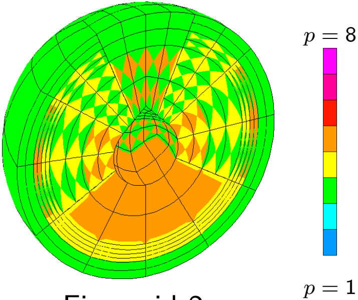


Coarse grid 3

← Error estimate and optimal hp-refinement



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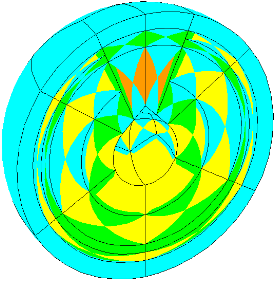
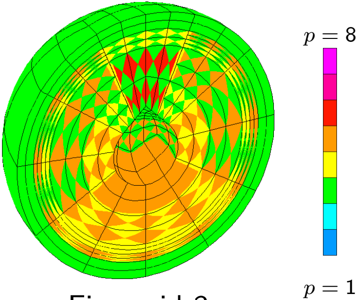


Coarse grid 3

Global

hp-refinement

→



Fine grid 3

The energy-driven mesh optimization algorithm

- Find optimal hp-refinements of the current coarse grid hp yielding the next coarse grid hp^{next} such that $(u = u_{h/2,p+1})$,

$$\frac{\|u - \Pi_{hp} u\| - \|u - \Pi_{hp^{next}} u\|}{N_{hp^{next}} - N_{hp}} \rightarrow \max$$

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- The algorithm reflects the logic of the projection-based interpolation and consists of three steps:
  - Determining optimal refinement of edges
  - Determining optimal refinement of faces
  - Determining optimal refinement of element interiors

  Each of the steps sets up initial conditions for the next step, limiting the number of cases to be considered.

Coupled Multiphysics Problems

- Simultaneous use of $H^1, H(\text{curl}), H(\text{div}), L^2$- conforming elements, C-preprocessing is used only to differentiate between the real and complex versions of the code.

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- Problem dependent code: computation of (unconstrained) element matrices, choice of norm.
- Problem independent code: description of geometry and multiphysics, constrained approximation, graphics, linear solvers, adaptivity, automatic h-,p- and hp-adaptivity.