

# Multiscale Analysis of Seismic Data Using the Seislet Transform

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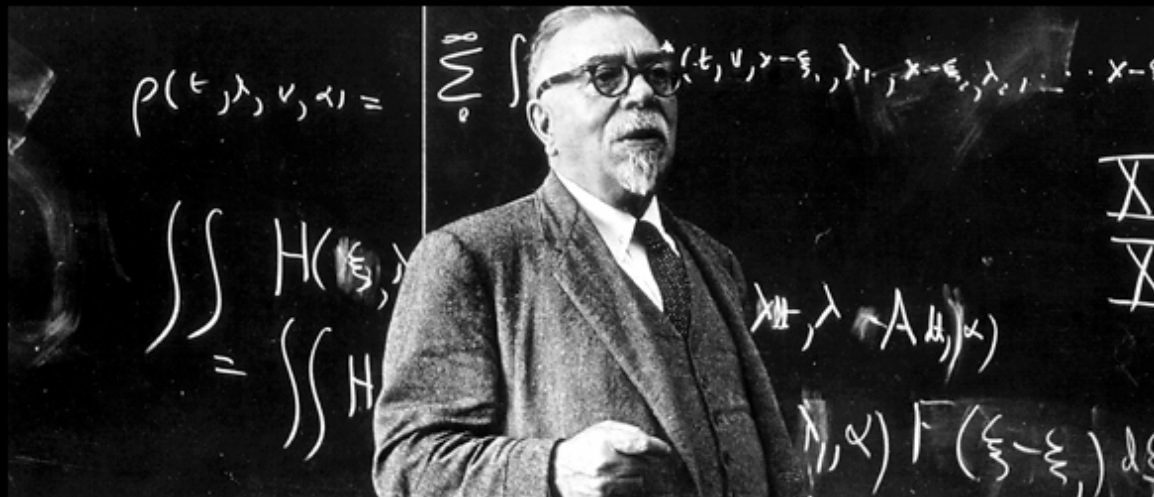
Joint work with Yang Liu

2008 Summer School on Multiscale Modeling and Analysis

# Exploration Geophysics



- ✓ **“Physics of the real world”**
- ✓ **Multiscale Earth**
- ✓ **Large scale computing**
  - ✓ **50 of the top 500 supercomputers**
- ✓ **Seismic imaging**
  - ✓ **Seismic data analysis**



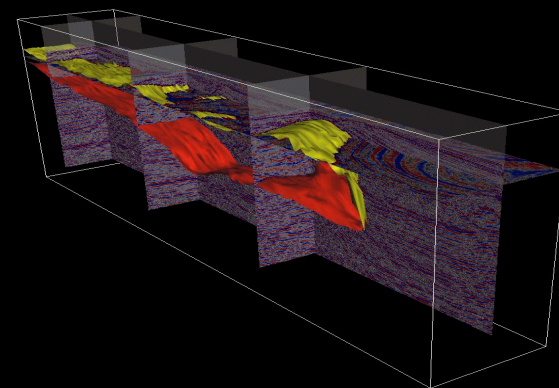
Norbert Wiener (© RLE/MIT)



Enders Robinson



Jon Claerbout



# Outline



- ✓ **Seismic data patterns**
  - ✓ local plane-wave prediction
- ✓ **From wavelets to seislets**
  - ✓ lifting scheme
- ✓ **From seislet transform to frame**
  - ✓ compressive sampling



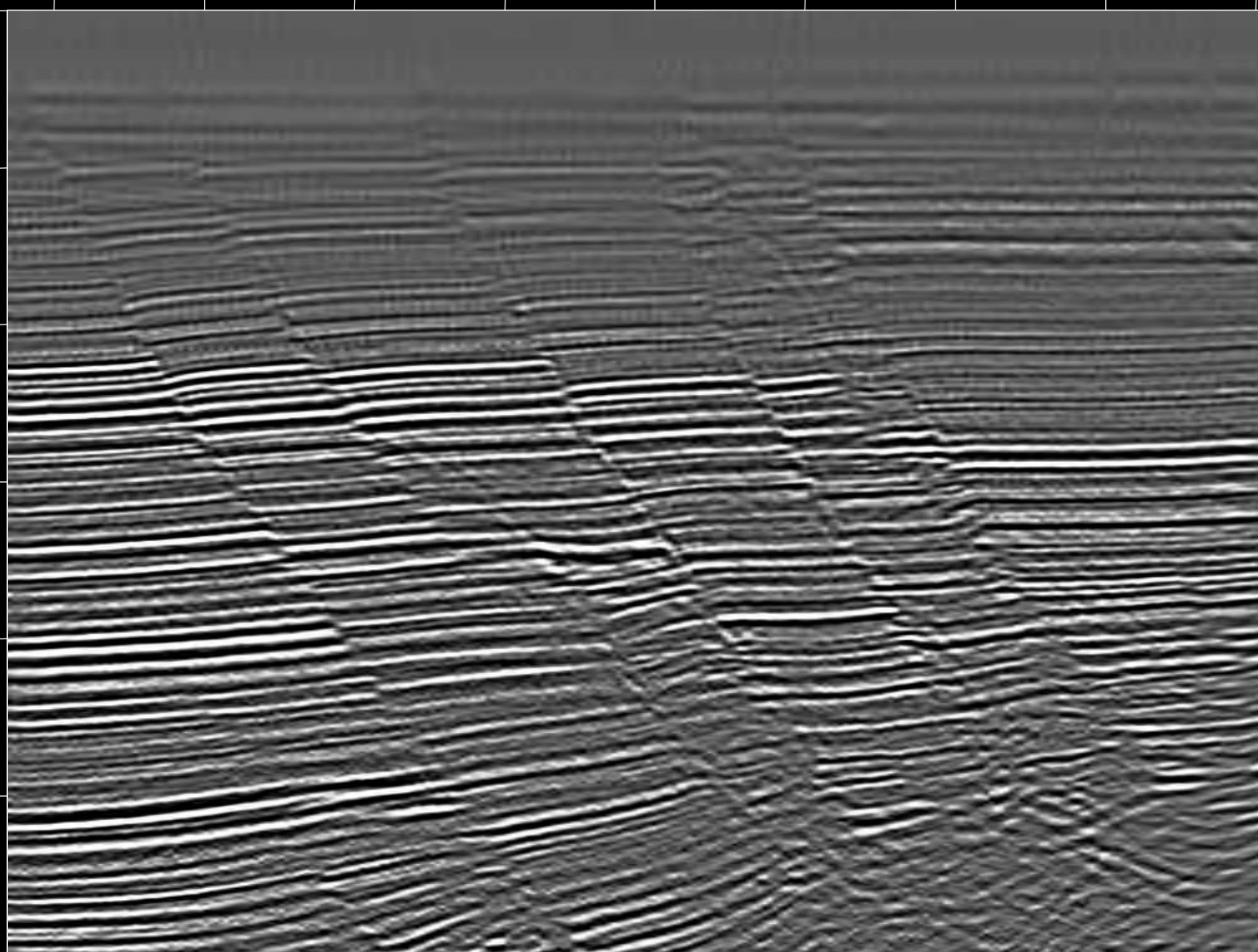
**Courtesy of Steven Arcone (US Army CRREL)**

Lateral (km)

8 9 10 11 12 13 14 15 16

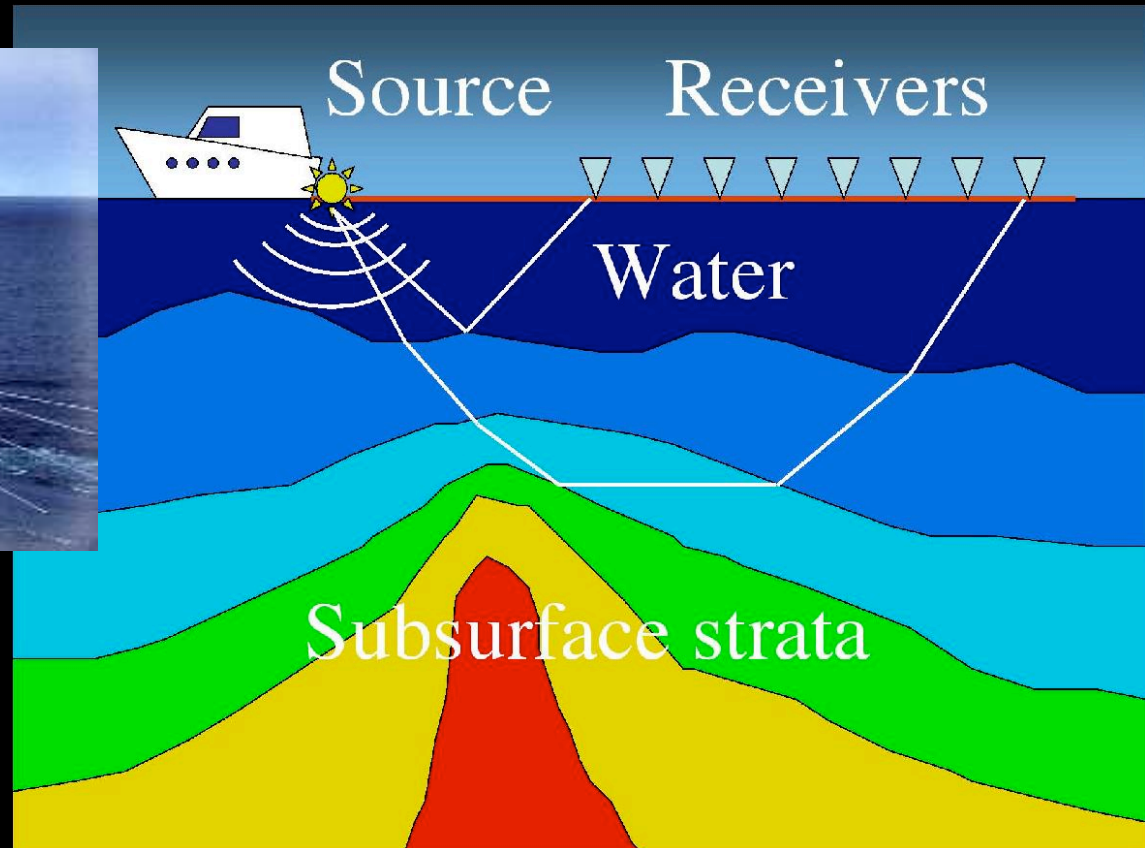
Time (s)

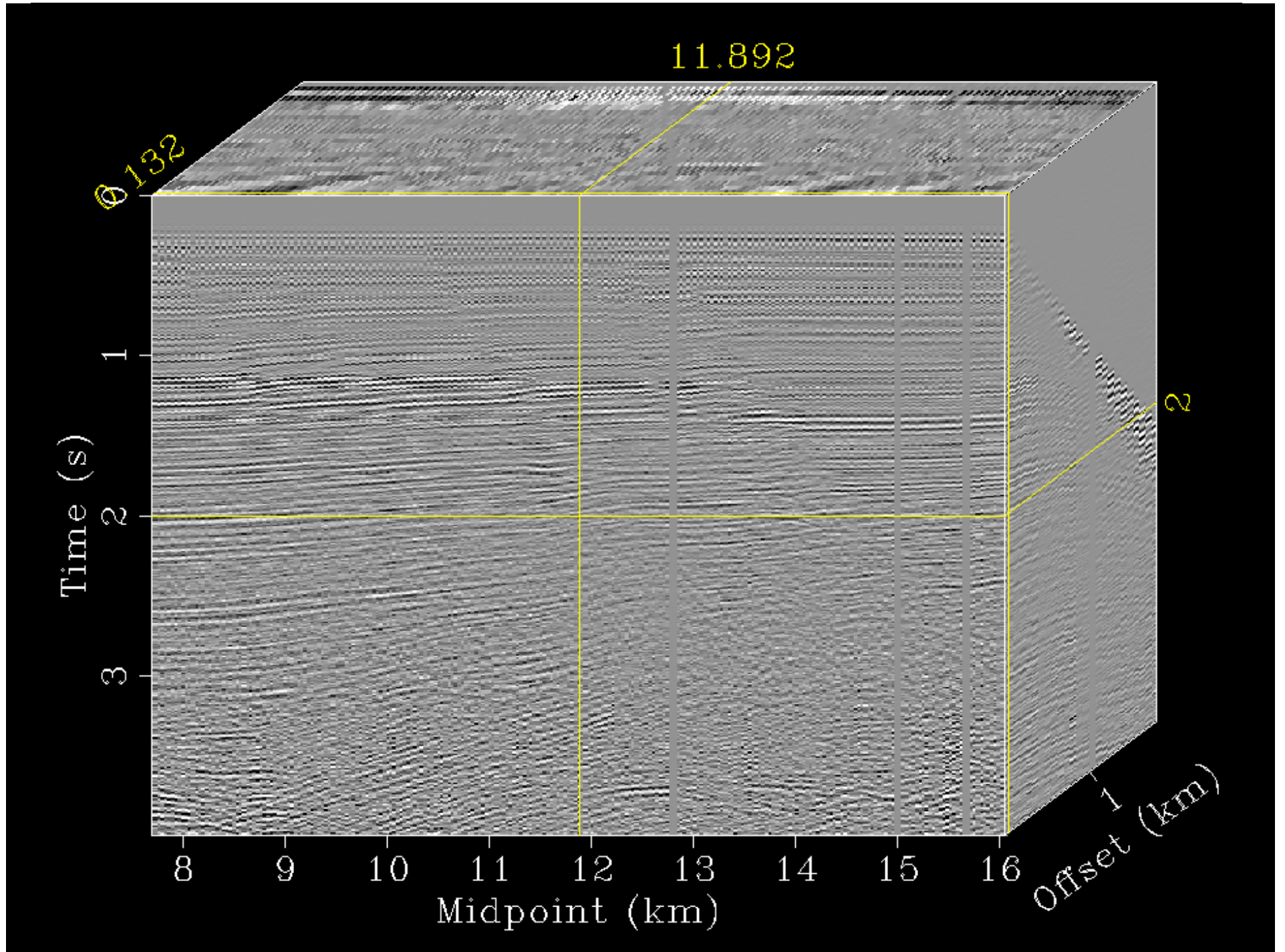
0  
0.5  
1  
1.5  
2  
2.5  
3



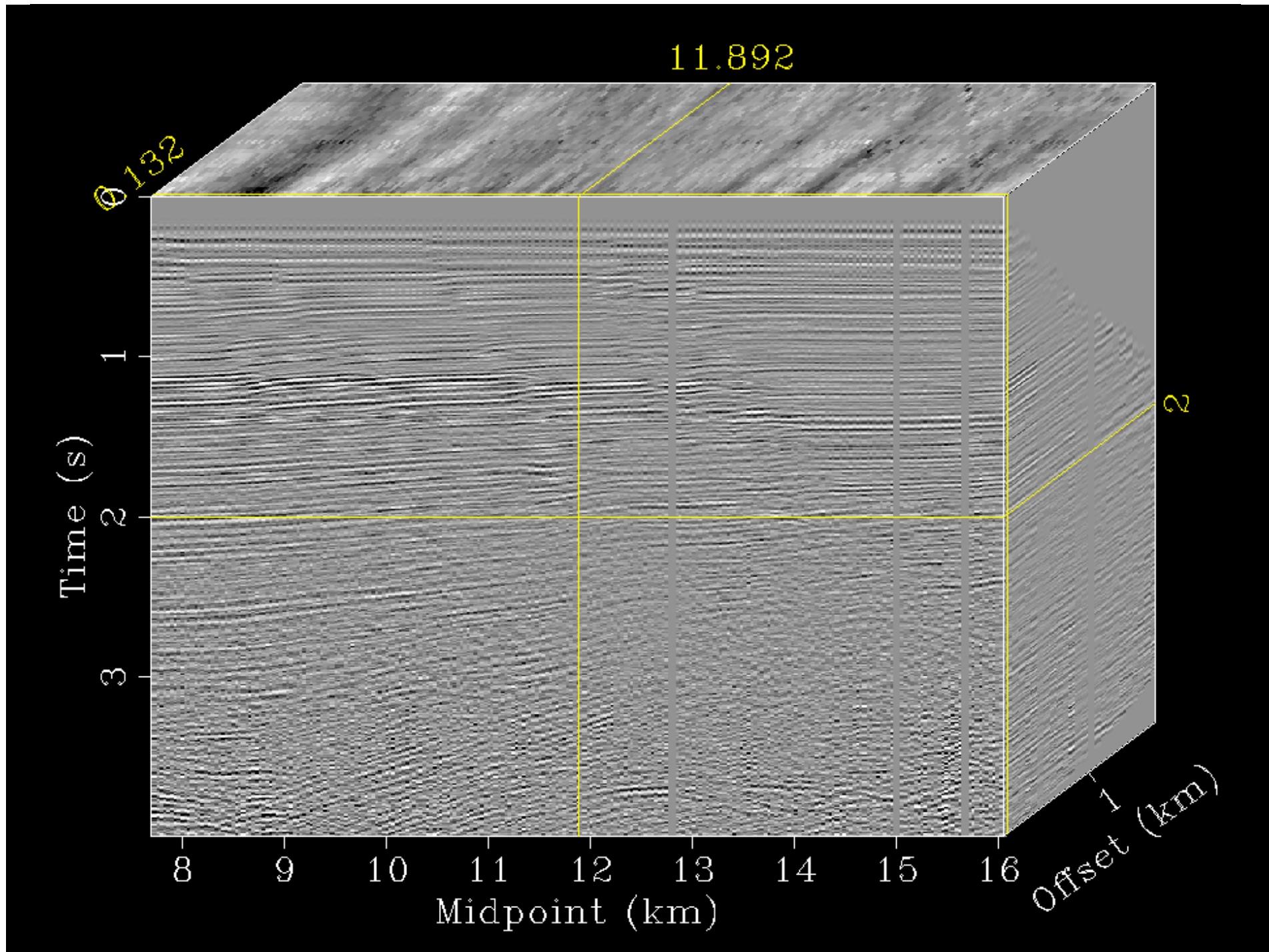
**Seismic Image**

# Seismic Reflection Method



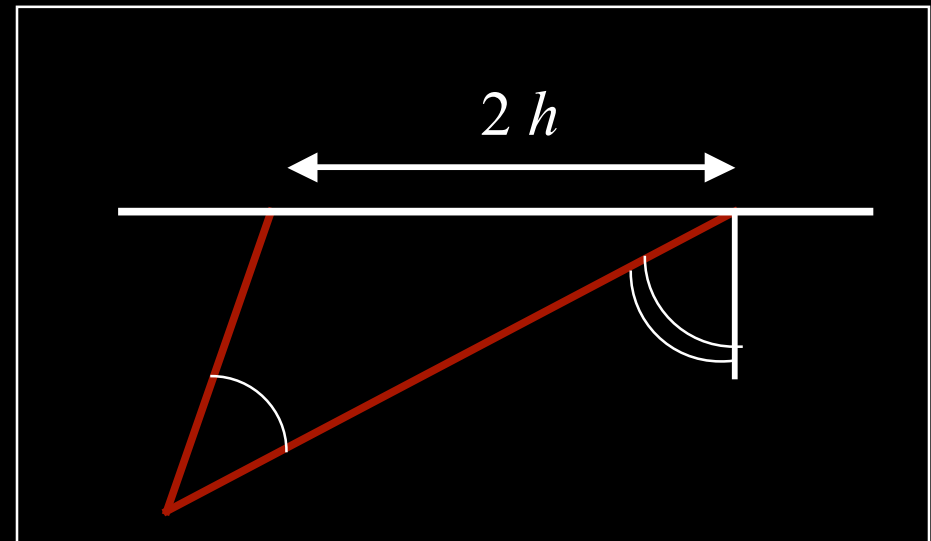






# Reflection Data Pattern

- ✓ Wave propagation in  $h$
- ✓ Offset continuation
- ✓ Linear hyperbolic PDE
- ✓ (Higginbotham, 1989)
- ✓ (Fomel, 1994; 2003)



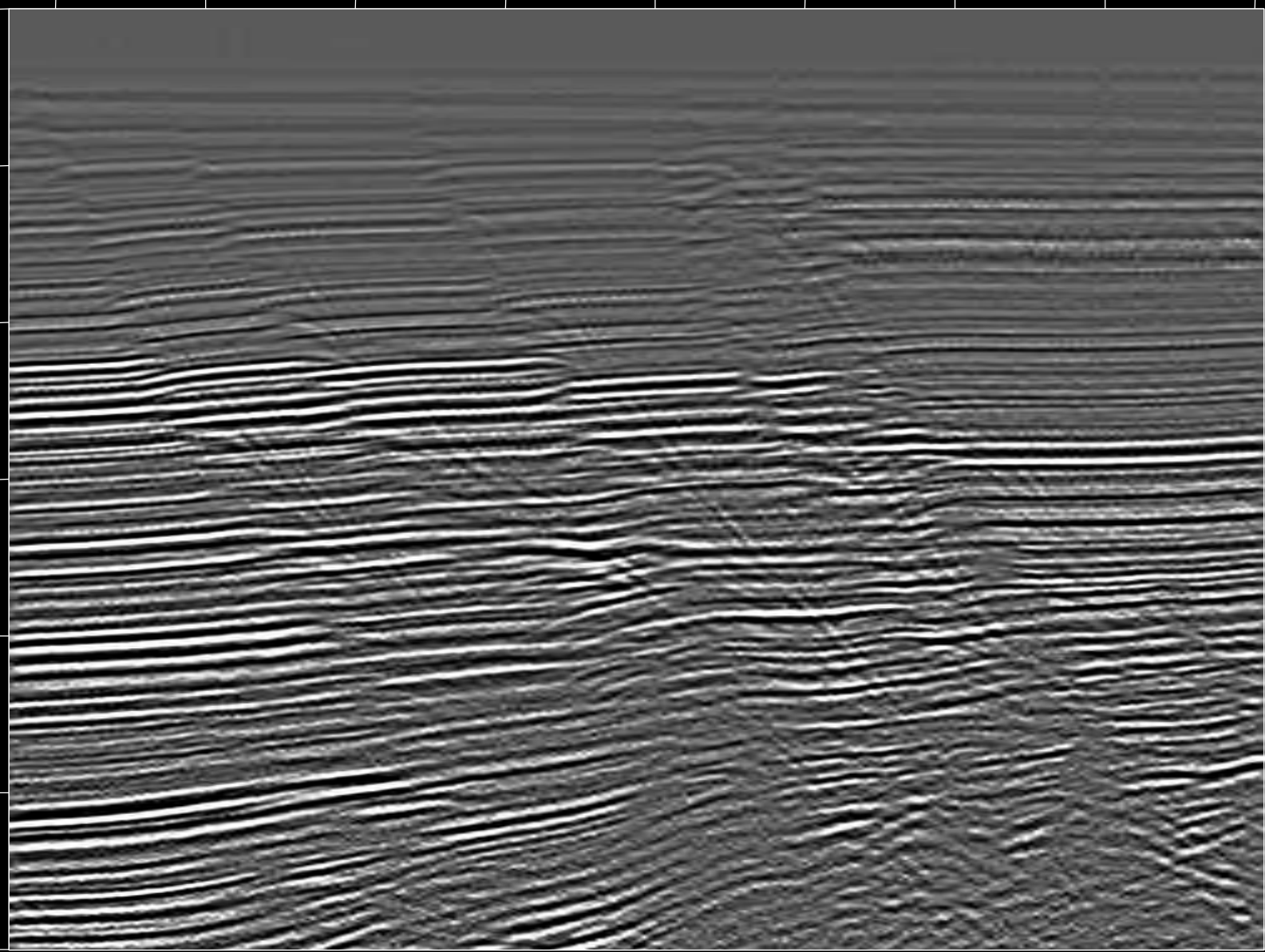
$$h \left( \frac{\partial^2 P}{\partial y^2} - \frac{\partial^2 P}{\partial h^2} \right) = t_n \frac{\partial^2 P}{\partial t_n \partial h}$$

Lateral (km)

8 9 10 11 12 13 14 15 16

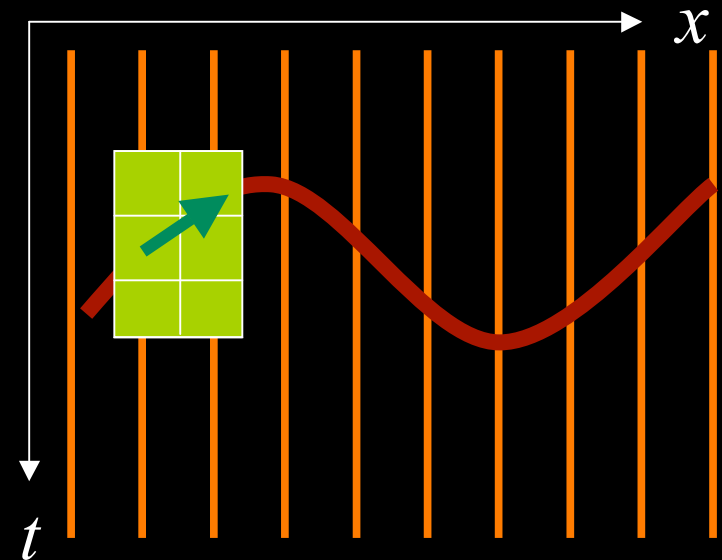
Time (s)

0  
0.5  
1  
1.5  
2  
2.5  
3



# Seismic Data Pattern

- ✓ Wave propagation in  $x$
- ✓ Plane-wave destruction
- ✓ Linear first-order PDE
- ✓ (Claerbout, 1992)
- ✓ (Fomel, 2002)



$$\frac{\partial I}{\partial x} + s(t, x) \frac{\partial I}{\partial t} = 0$$

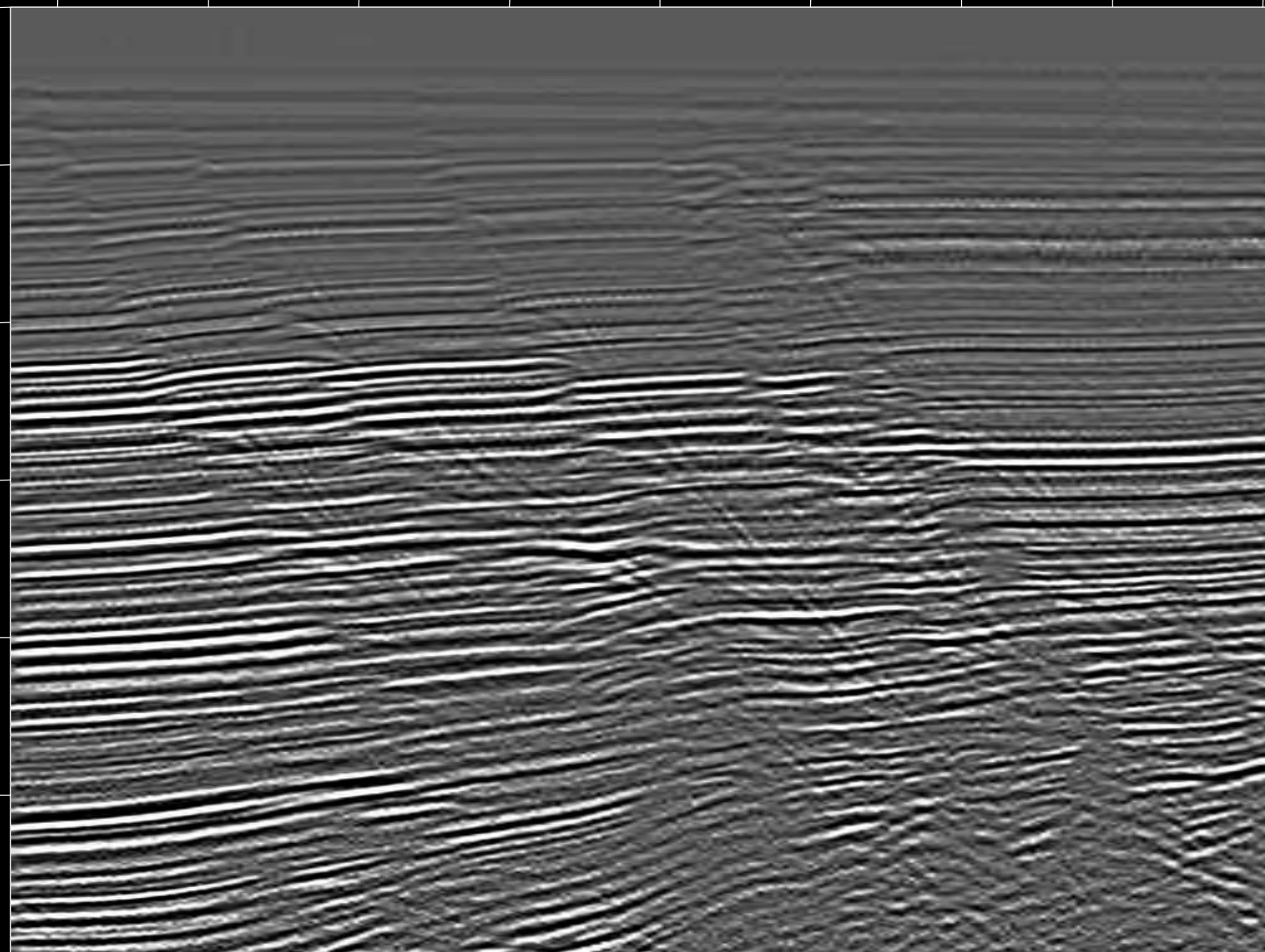


Lateral (km)

8 9 10 11 12 13 14 15 16

Time (s)

0  
0.5  
1  
1.5  
2  
2.5  
3

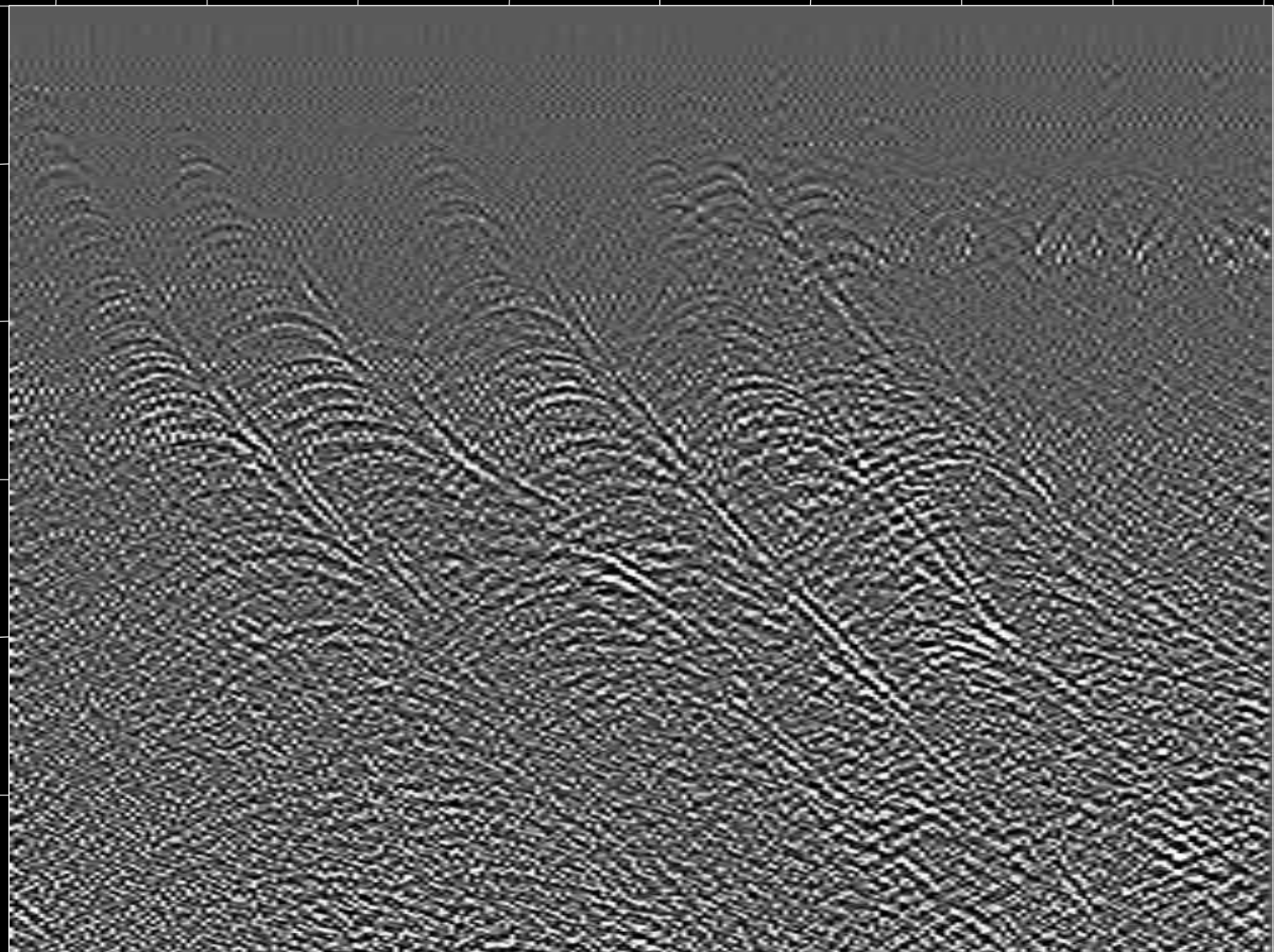


Lateral (km)

8 9 10 11 12 13 14 15 16

Time (s)

0  
0.5  
1  
1.5  
2  
2.5  
3



# Outline



- ✓ **Seismic data patterns**
  - ✓ local plane-wave prediction
- ✓ **From wavelets to seislets**
  - ✓ lifting scheme
- ✓ **From seislet transform to frame**
  - ✓ compressive sampling

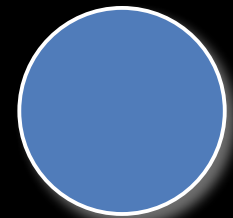


# \*let Transforms



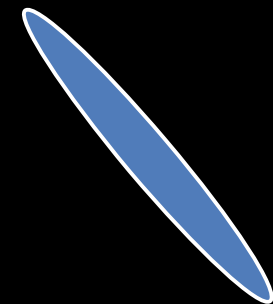
## ✓ Wavelets

- ✓ From French “ondelette” (Morlet, 1975)
- ✓ Continuous and discrete
- ✓ Multiscale



## ✓ Xlets

- ✓ Bandelets, beamlets, contourlets, curvelets, directionlets, ...
- ✓ Seislets



# Review of Wavelets: Haar

- ✓ Take digital data:

6	6	4	6	10	6	8	6
---	---	---	---	----	---	---	---

- ✓ Divide into pairs:

6	6	4	6	10	6	8	6
---	---	---	---	----	---	---	---

- ✓ Compute averages and differences:

6	0	5	-2	8	4	7	2
---	---	---	----	---	---	---	---

# Review of Wavelets: Haar

- ✓ New data are averages:

6	5	8	7	0	-2	4	2
---	---	---	---	---	----	---	---

- ✓ Divide into pairs:

6	5	8	7	0	-2	4	2
---	---	---	---	---	----	---	---

- ✓ Compute averages and differences:

5.5	1	7.5	1	0	-2	4	2
-----	---	-----	---	---	----	---	---

# Review of Wavelets: Haar

- ✓ New data are averages:

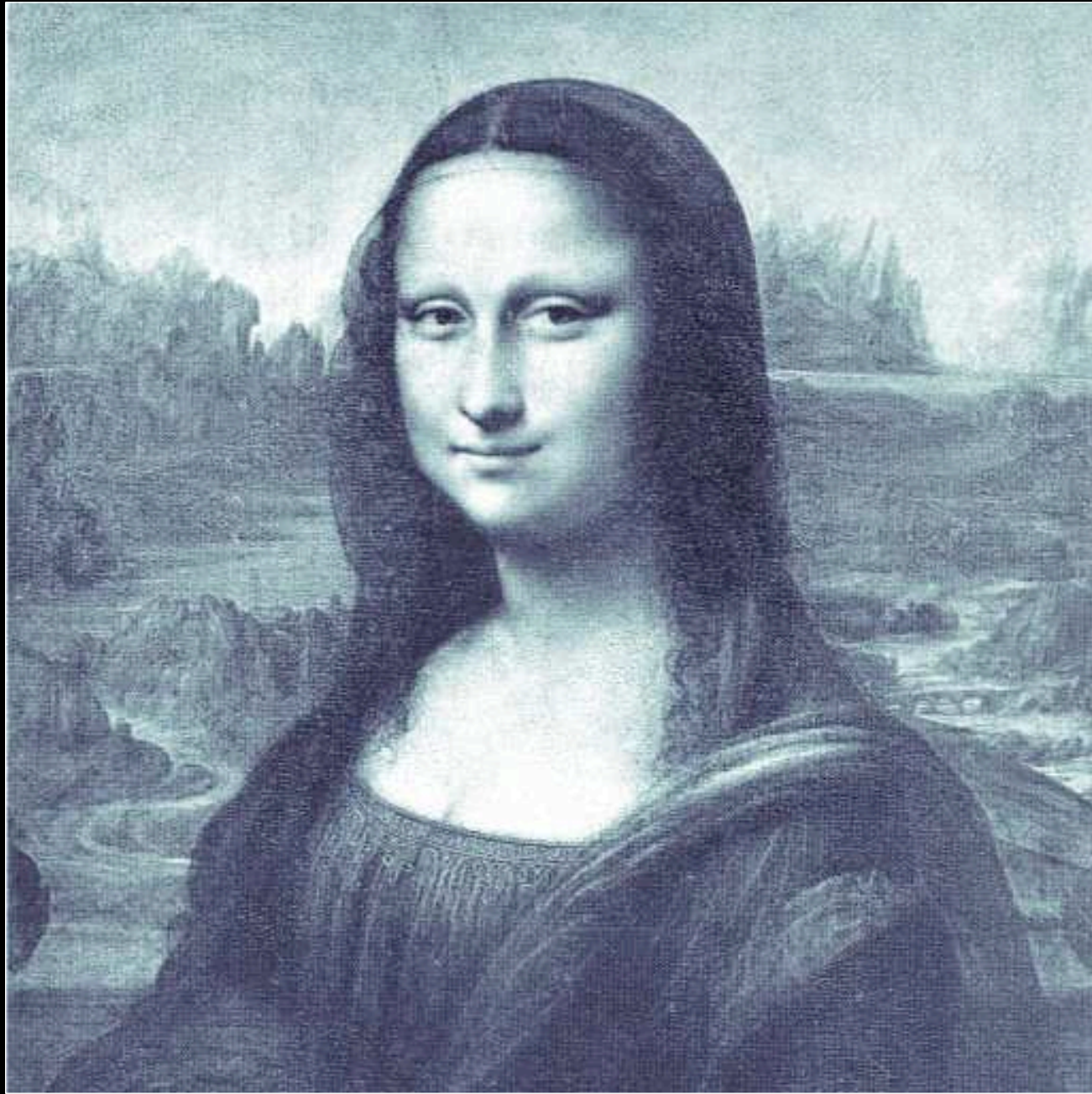
5.5	7.5	1	1	0	-2	4	2
-----	-----	---	---	---	----	---	---

- ✓ Divide into pairs:

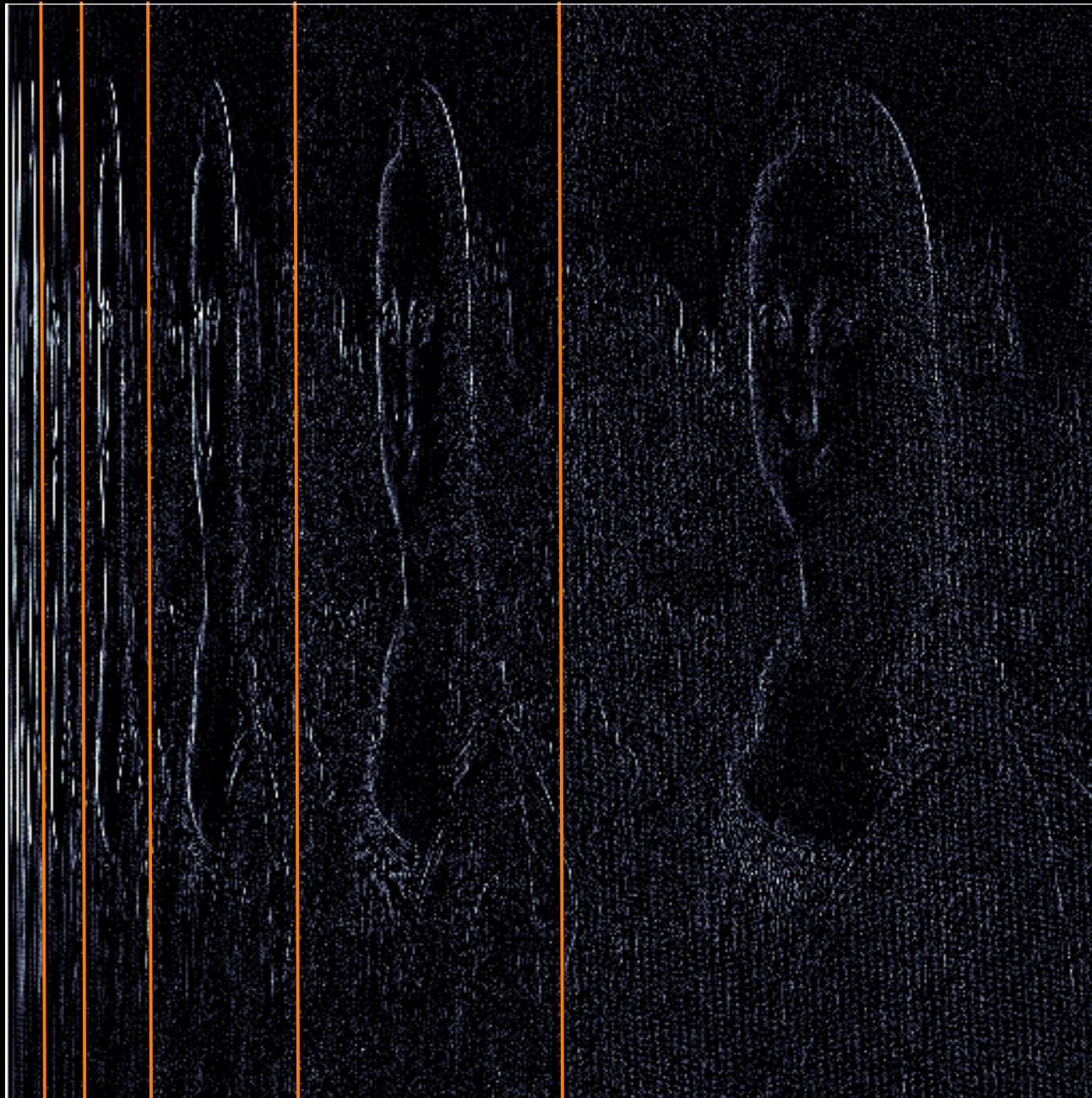
5.5	7.5	1	1	0	-2	4	2
-----	-----	---	---	---	----	---	---

- ✓ Compute averages and differences:

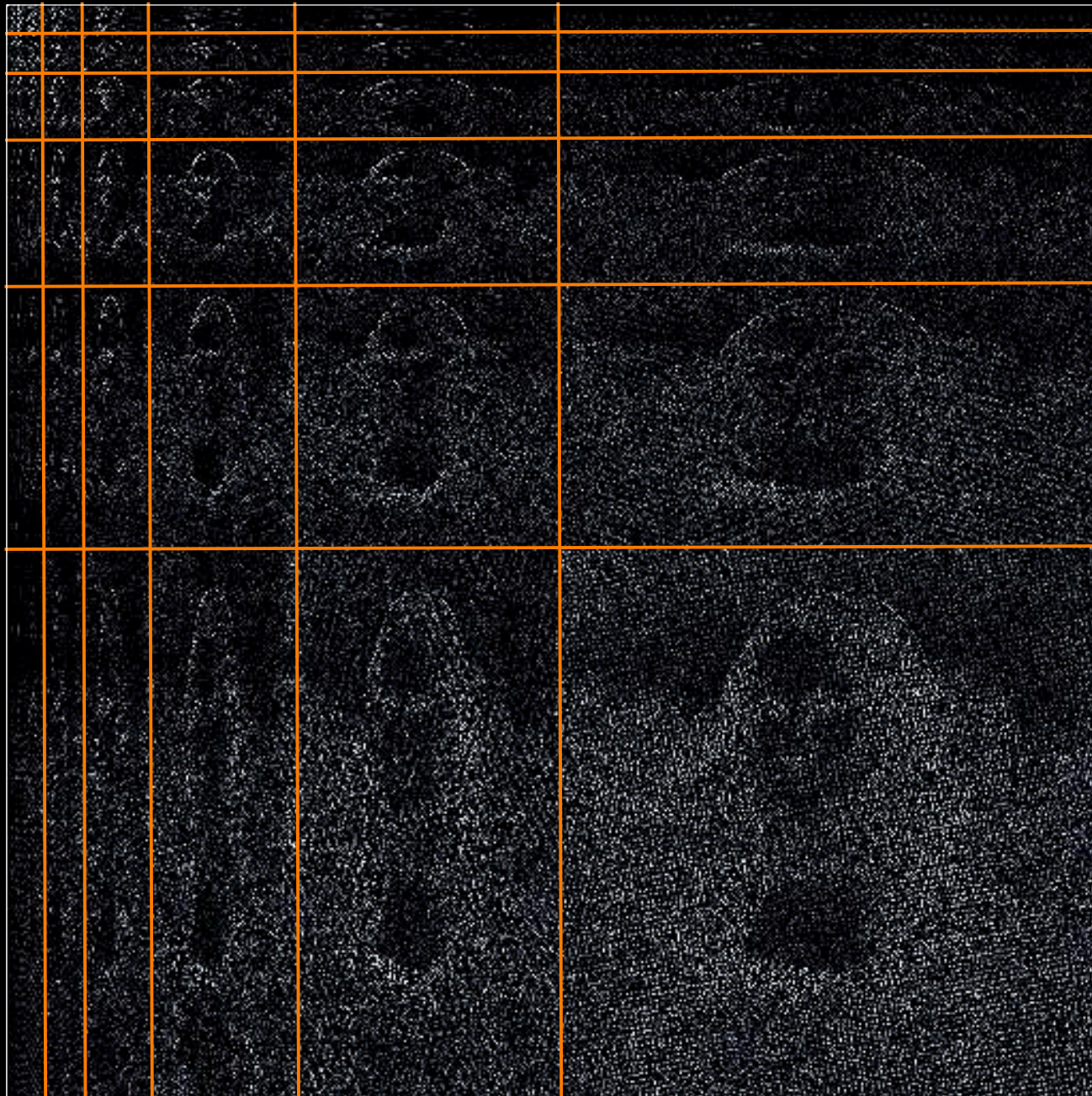
6	-2	1	1	0	-2	4	2
---	----	---	---	---	----	---	---



**Mona Lisa**



**1-D Haar Transform**



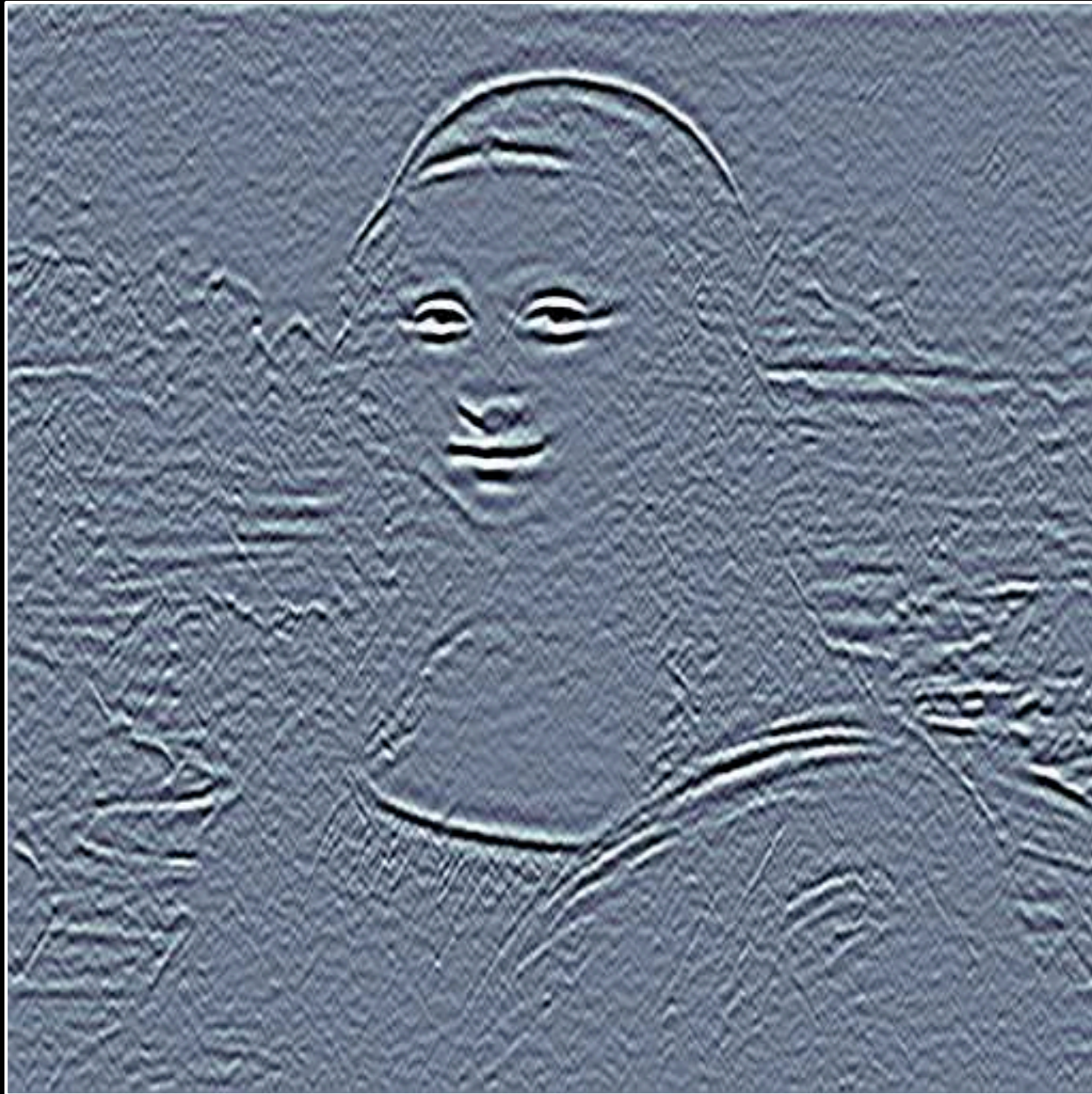
**2-D Haar Transform**

# Digital Wavelet Transform



- ✓ **Exactly invertible**
- ✓  **$O(N)$  operations**
  - ✓ **faster than FFT**
- ✓ **Defines a basis**
- ✓ **Multiple scales**





**Seismic Mona Lisa**

# Haar Lifting Scheme

- ✓ **Forward transform**

- ✓ split **s** into **o** and **e**

$$d = o - e$$

- ✓ compute **s** and **d**

$$s = (o + e)/2 = e + d/2$$

- at the next scale

- ✓ **Inverse transform**

- ✓ compute **o** and **e**

- at the previous scale

$$e = s - d/2$$

- ✓ merge **o** and **e** into **s**

$$o = d + e$$

# General Lifting Scheme

(Sweldens and Schröder, "Building your own wavelets at home," 1996)

## ✓ Forward transform

- ✓ split **s** into **o** and **e**
- ✓ compute **s** and **d**  
at the next scale

$$\begin{aligned}d &= o - P[e] \\s &= e + U[d]\end{aligned}$$

## ✓ Inverse transform

- ✓ compute **o** and **e**  
at the previous scale
- ✓ merge **o** and **e** into **s**

$$\begin{aligned}e &= s - U[d] \\o &= d + P[e]\end{aligned}$$

# Building Seislets at Home



- ✓ **Think of seismic data as a collection of records**
- ✓ **Pick your favorite digital wavelet transform**
- ✓ **Design the prediction operator to predict individual records from their neighbors**
- ✓ **Design the update operator to carry main data characteristics to the next scale level**

# 1-D Seislet Transform

- ✓ Haar predicts a constant

- ✓  $[1, -1] * [c, c, c, \dots] = 0$

- ✓  $(1 - Z) (1 + Z + Z^2 + \dots) = 0$

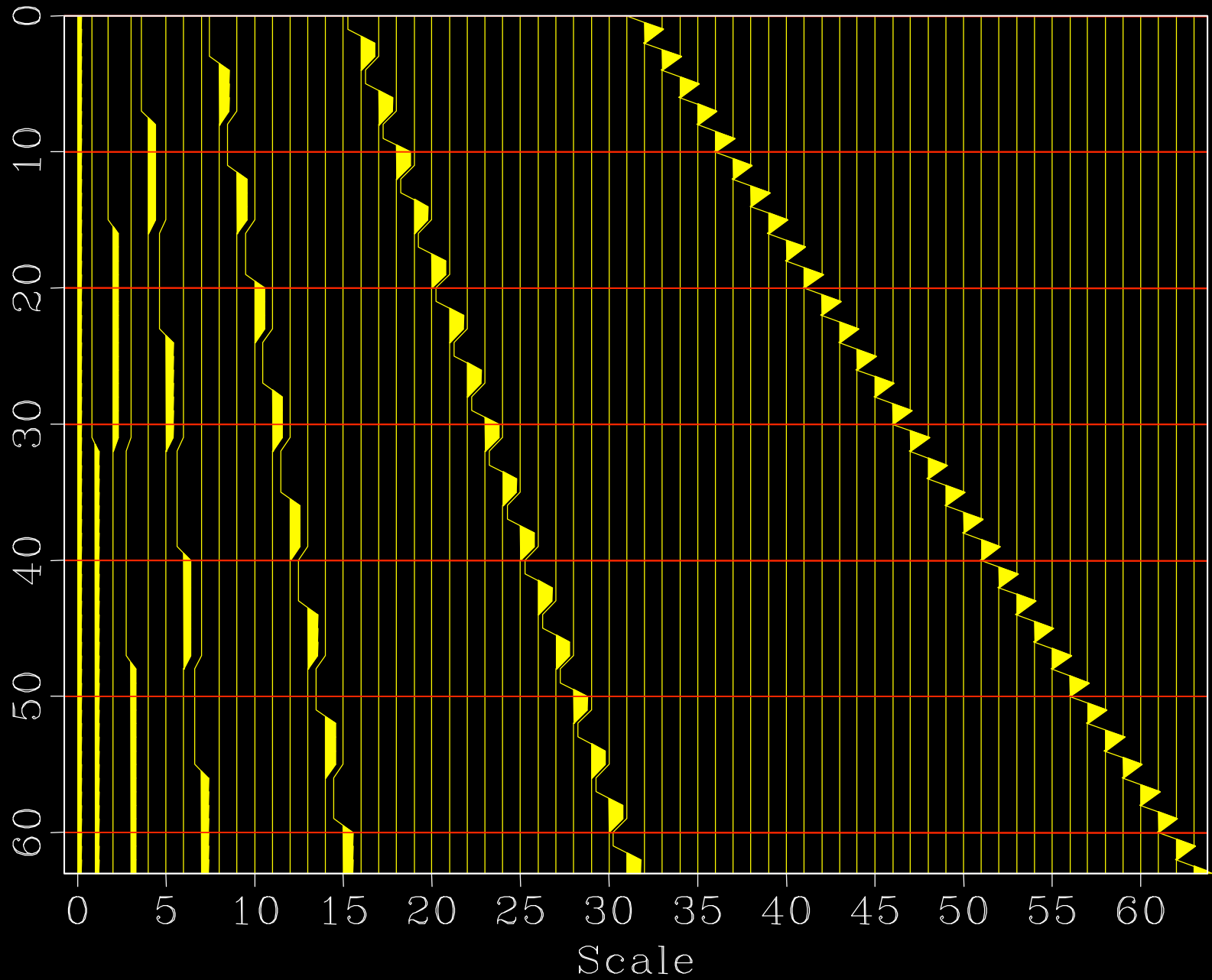
- ✓ Predict an exponential or sinusoid

- ✓  $[1, -a] * [c, c a, c a^2, \dots] = 0$

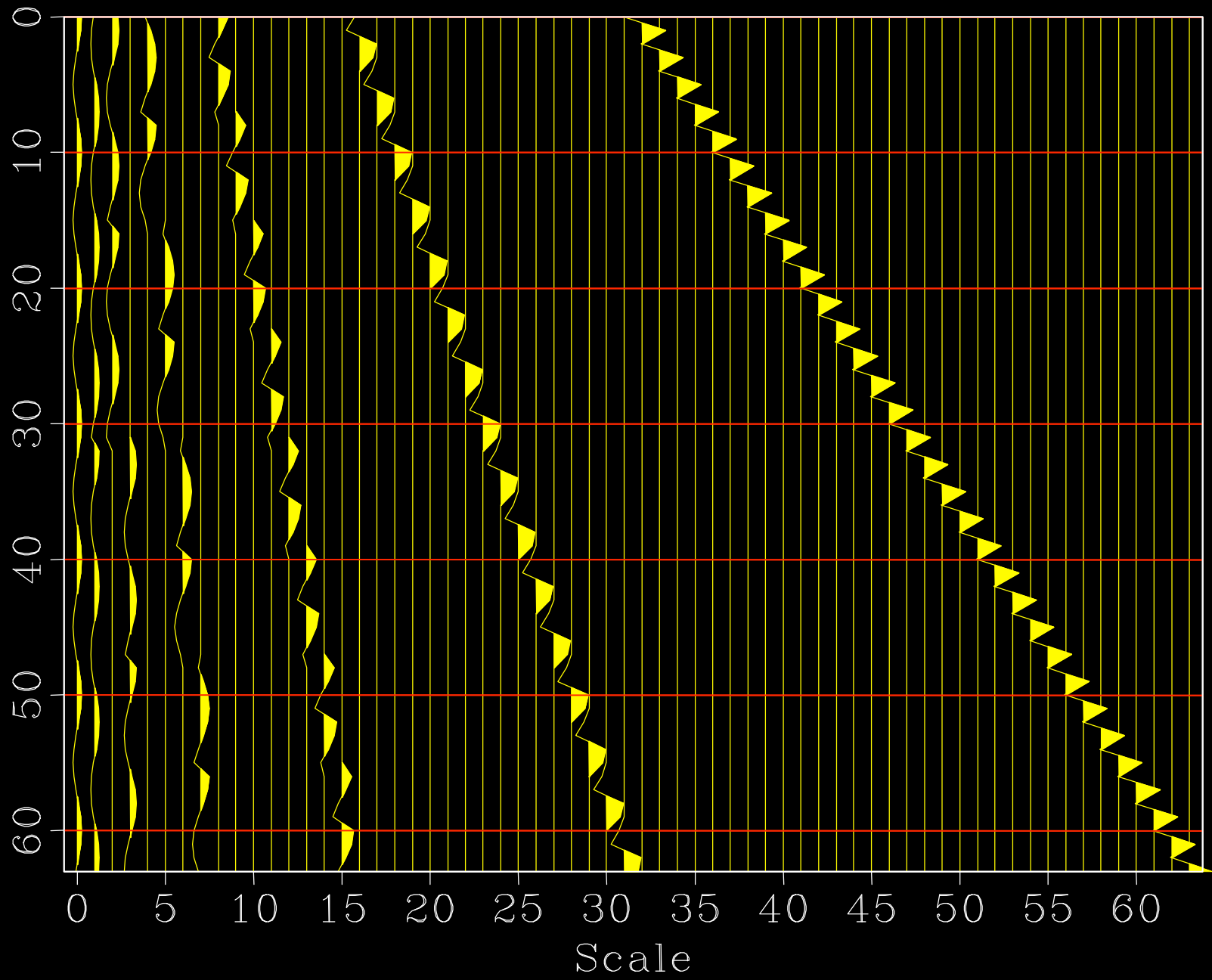
- ✓  $(1 - Z / Z_0) (1 + Z / Z_0 + (Z / Z_0)^2 + \dots) = 0$

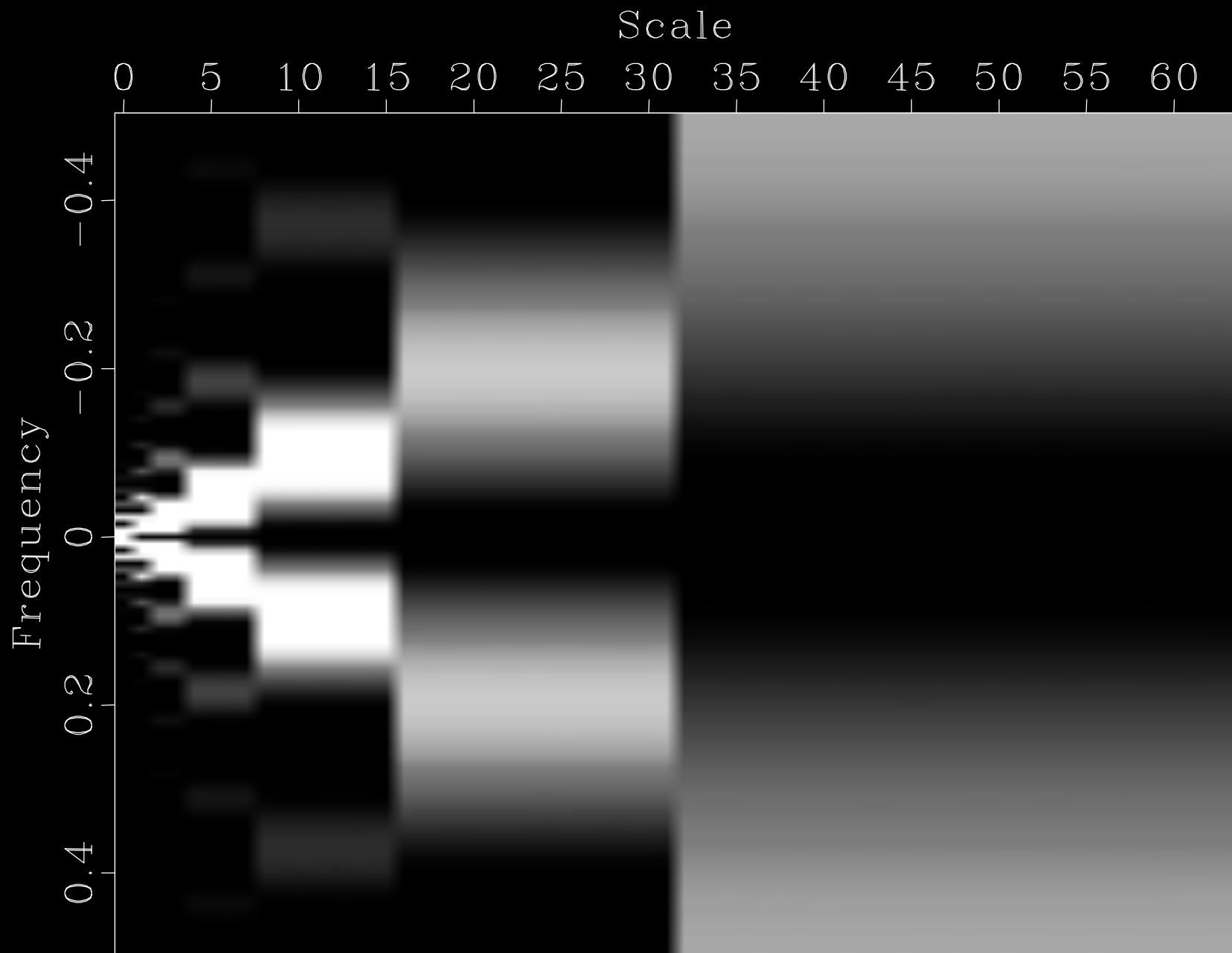
- ✓  $Z_0 = \exp(i \omega_0)$

# 1-D Wavelets



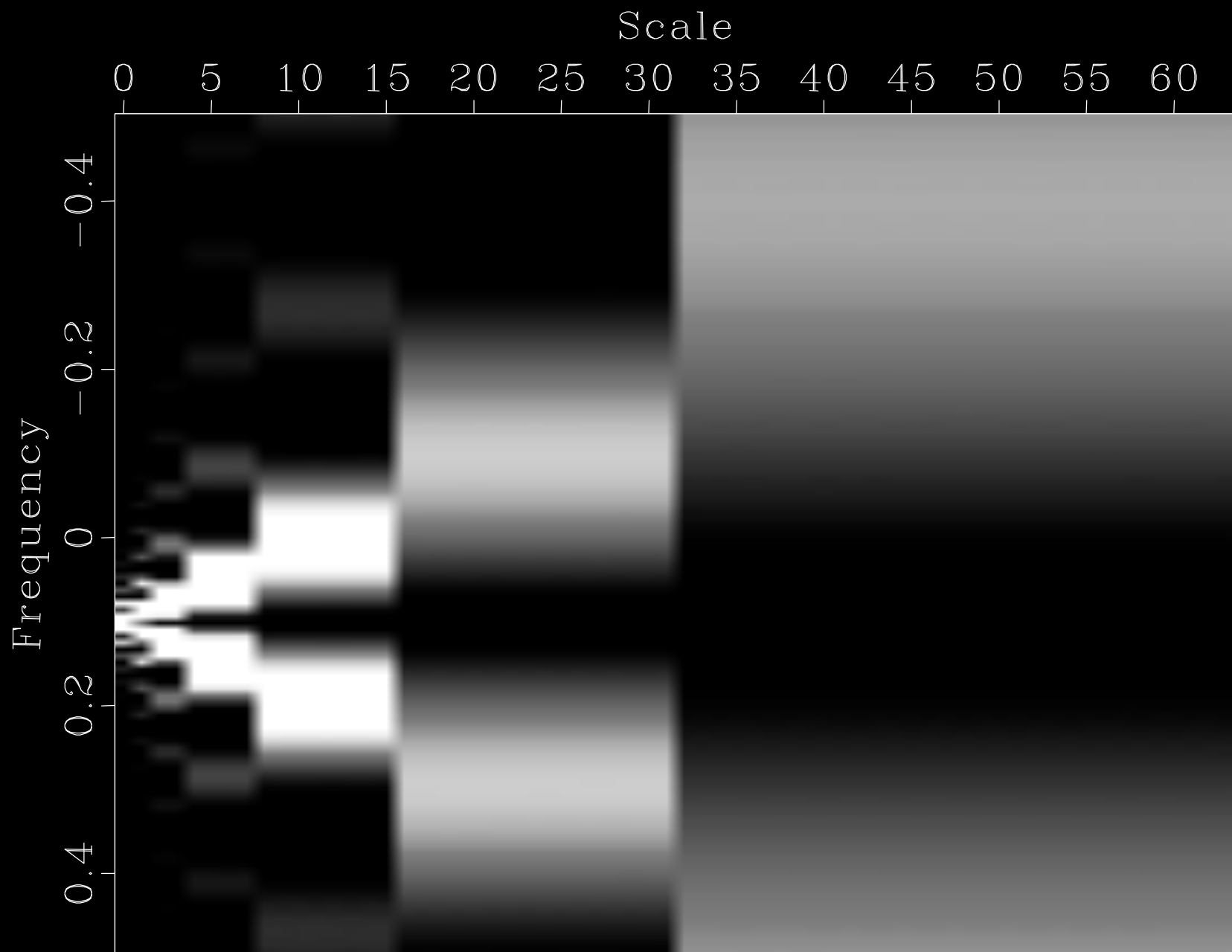
# 1-D Seislets





1-D Wavelets



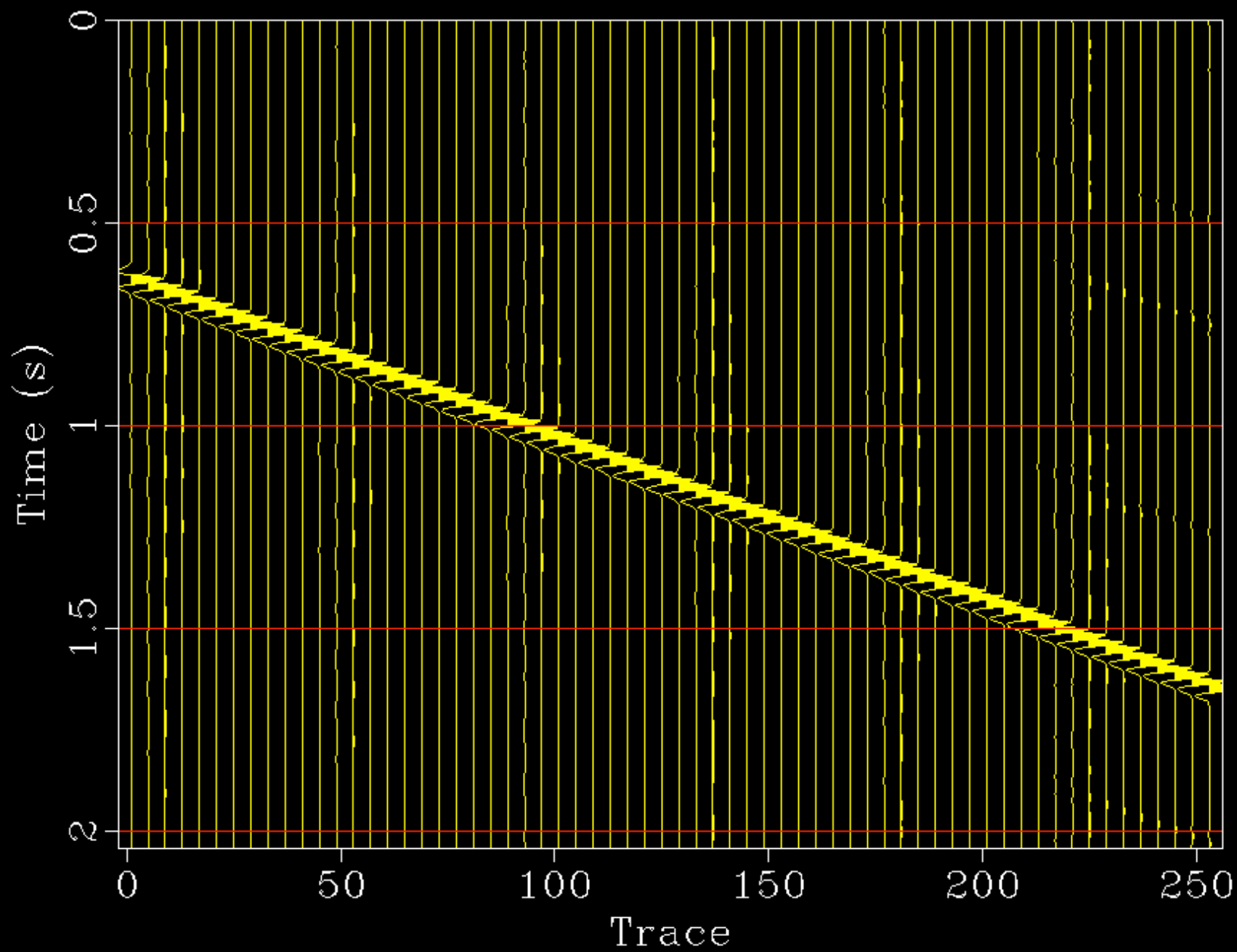


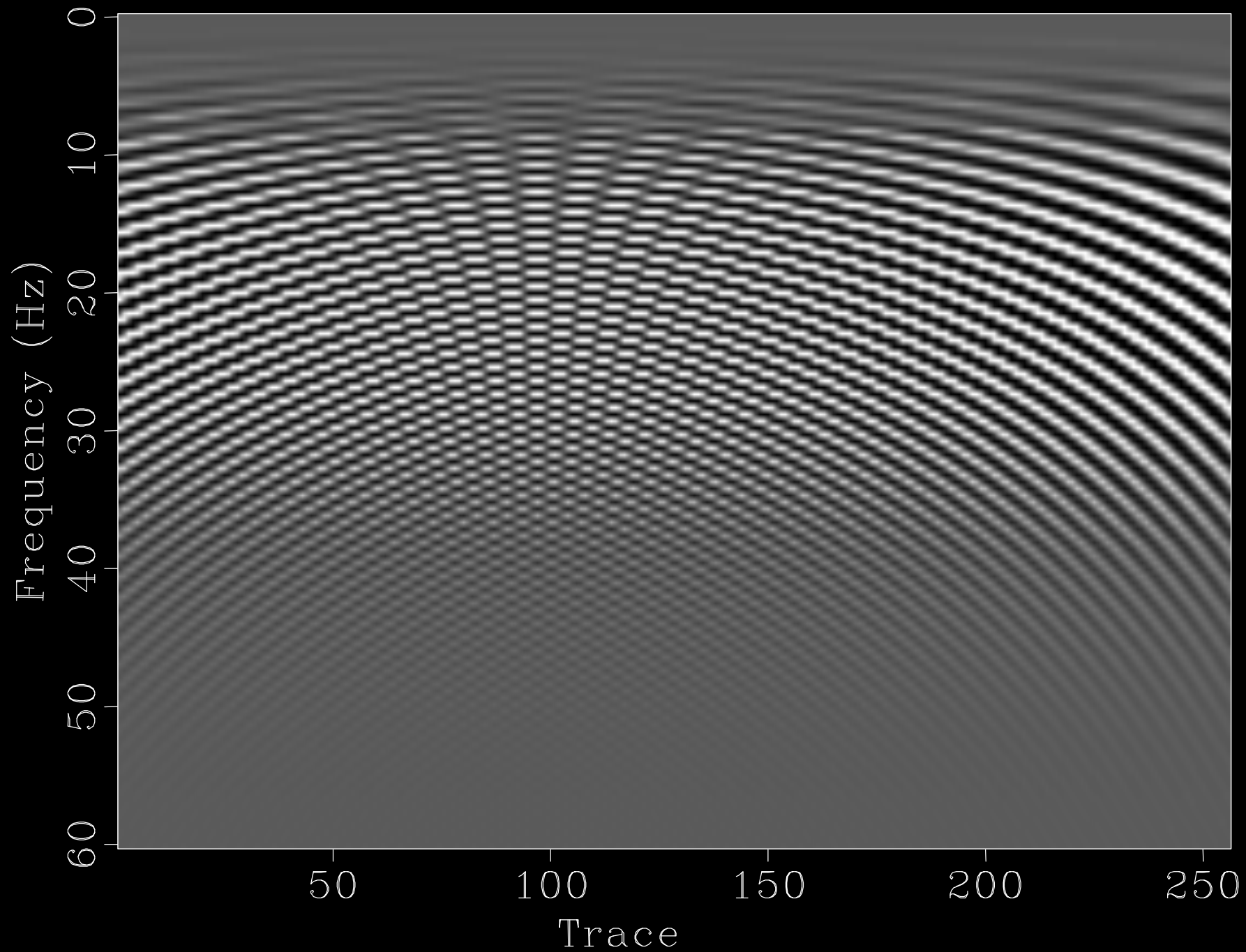
1-D Seislets

# 1-D Seislets in F-X domain

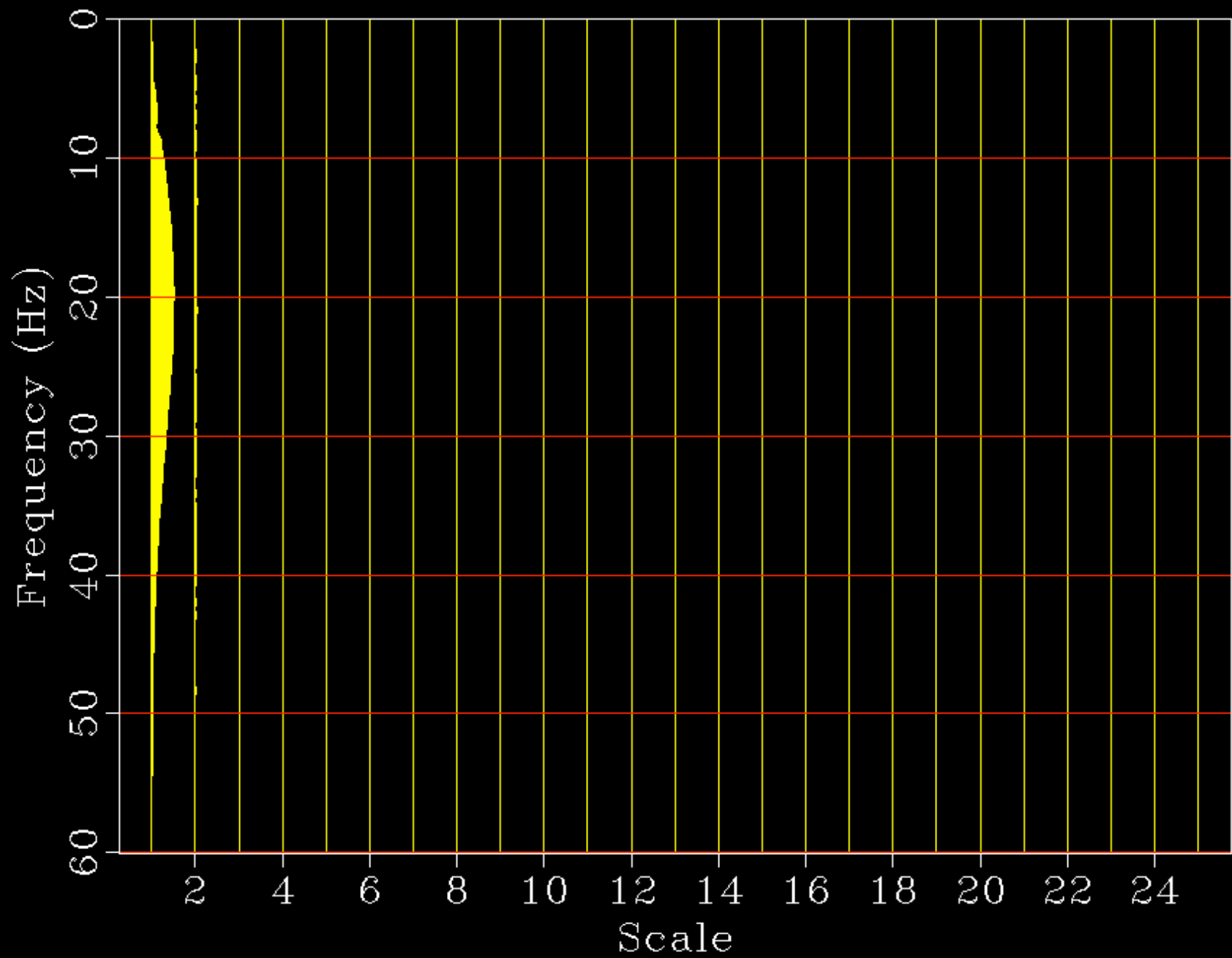
- ✓ From T-X to F-X
  - ✓ Fourier transform in time
  - ✓ planes to sinusoids
- ✓ 1-D seislet transform
  - ✓ frequency slices

# Plane 3





# Component 3

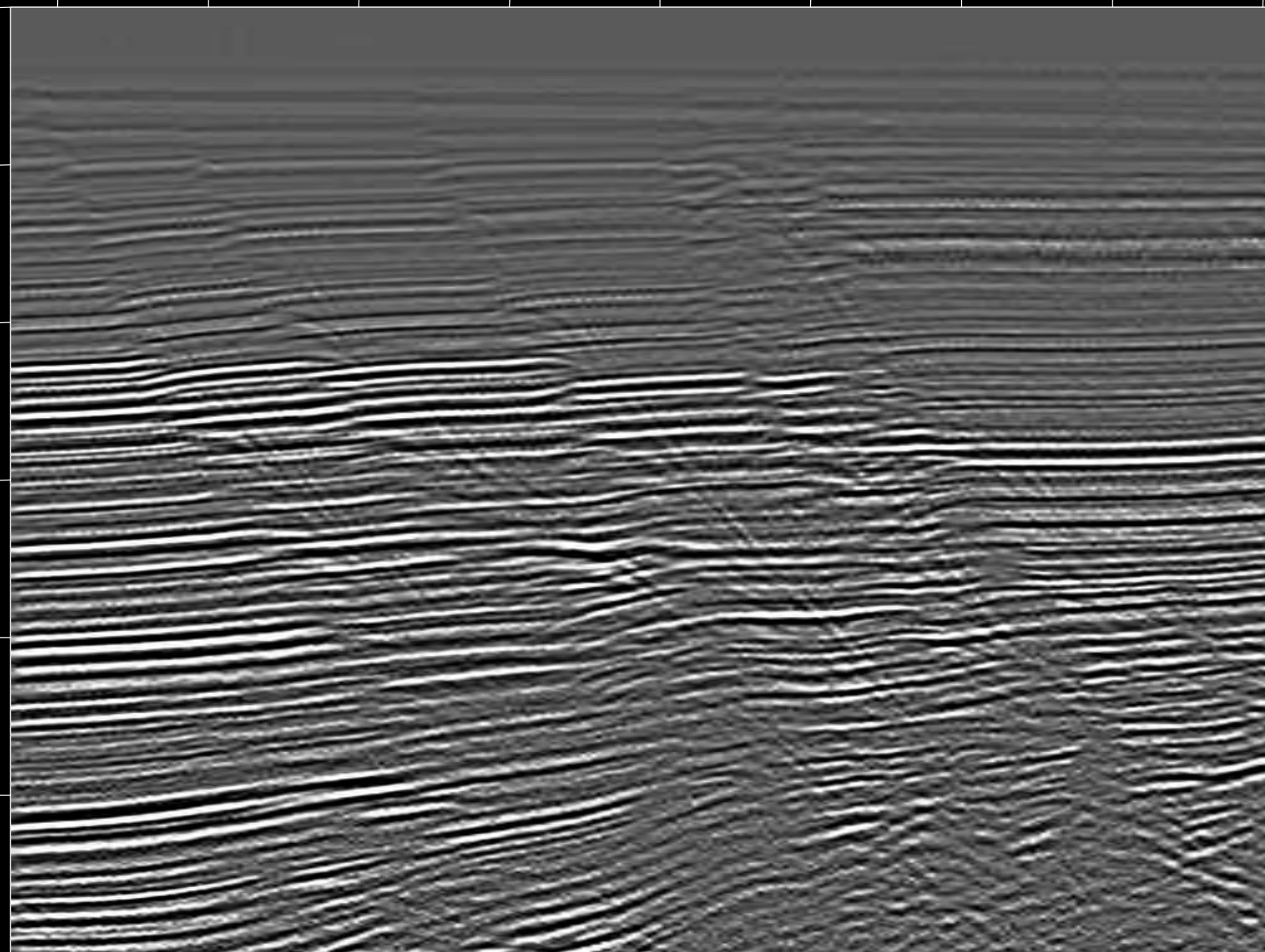


Lateral (km)

8 9 10 11 12 13 14 15 16

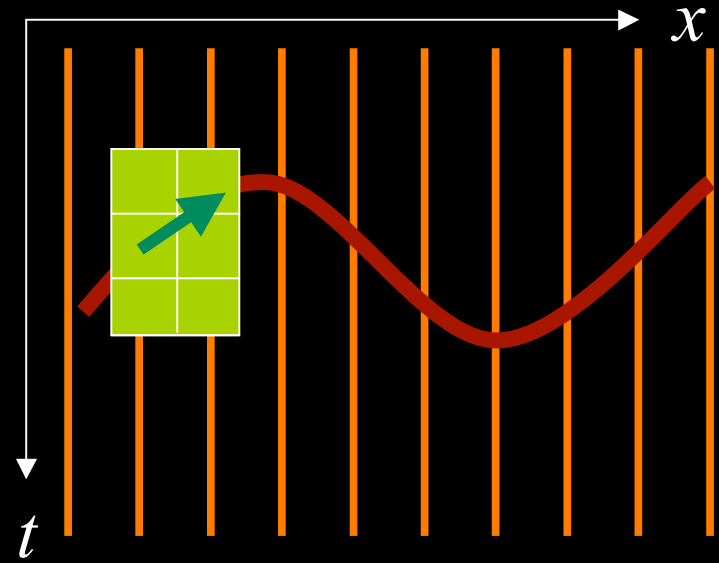
Time (s)

0  
0.5  
1  
1.5  
2  
2.5  
3



# Seismic Data Pattern

- ✓ Wave propagation in  $x$
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- ✓ (Fomel, 2002)



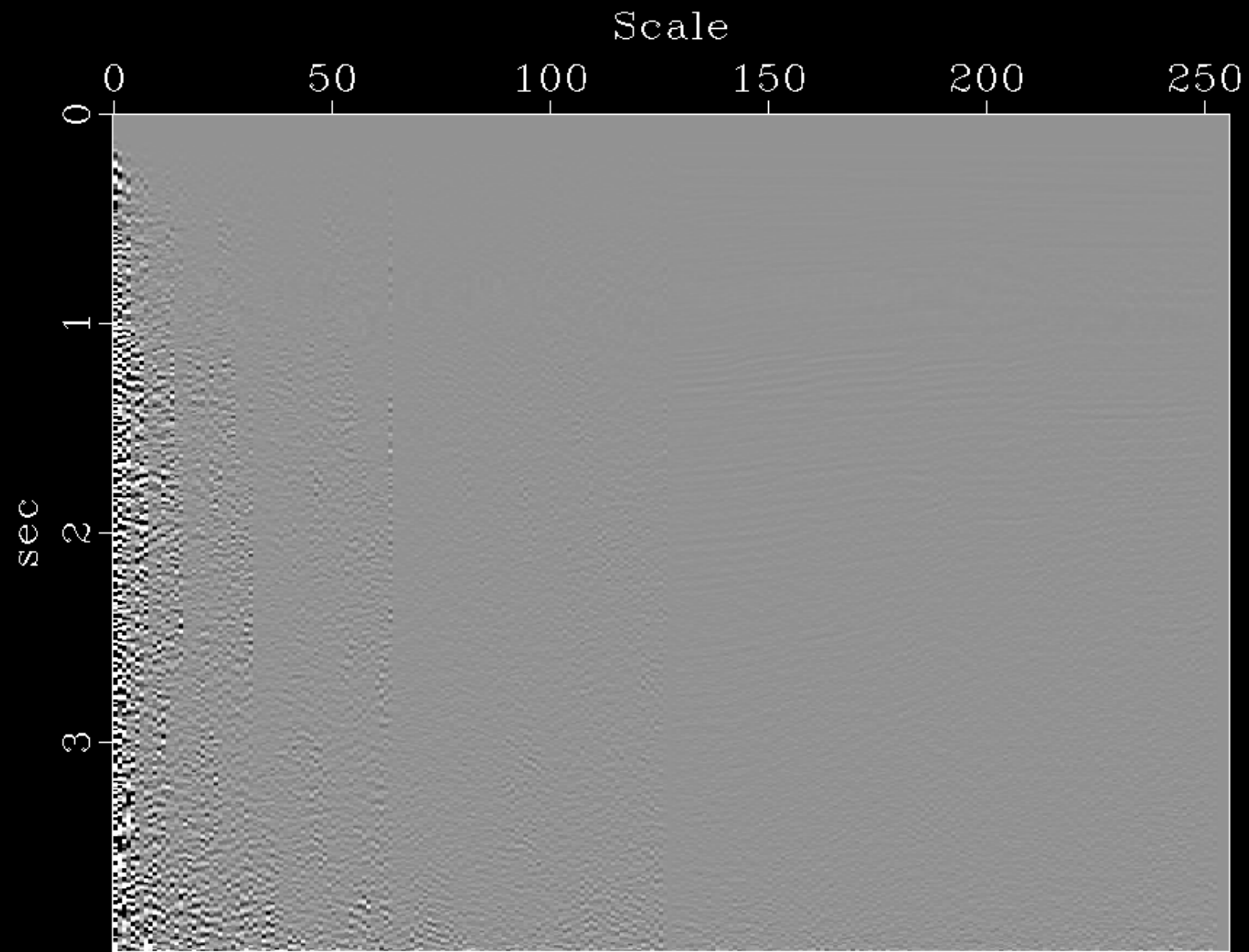
$$\frac{\partial I}{\partial x} + s(t, x) \frac{\partial I}{\partial t} = 0$$

# 2-D Seislet Transform

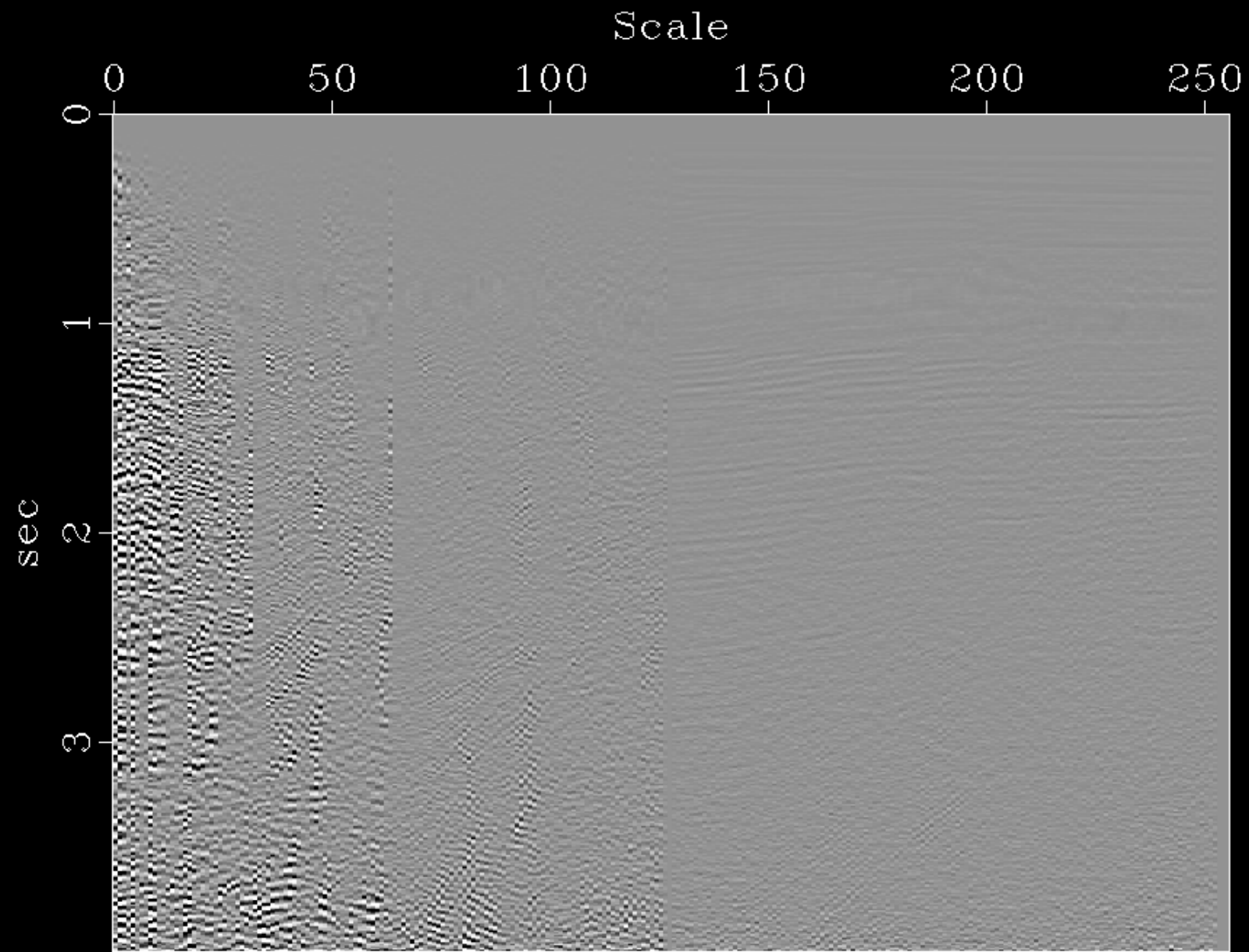
- ✓ View seismic data as a collection of traces
- ✓ Define trace prediction by local plane-wave propagation along dominant slopes
- ✓ Estimate local slopes by minimizing prediction error
- ✓ Combine predictions and updates from different scales into a lifting scheme



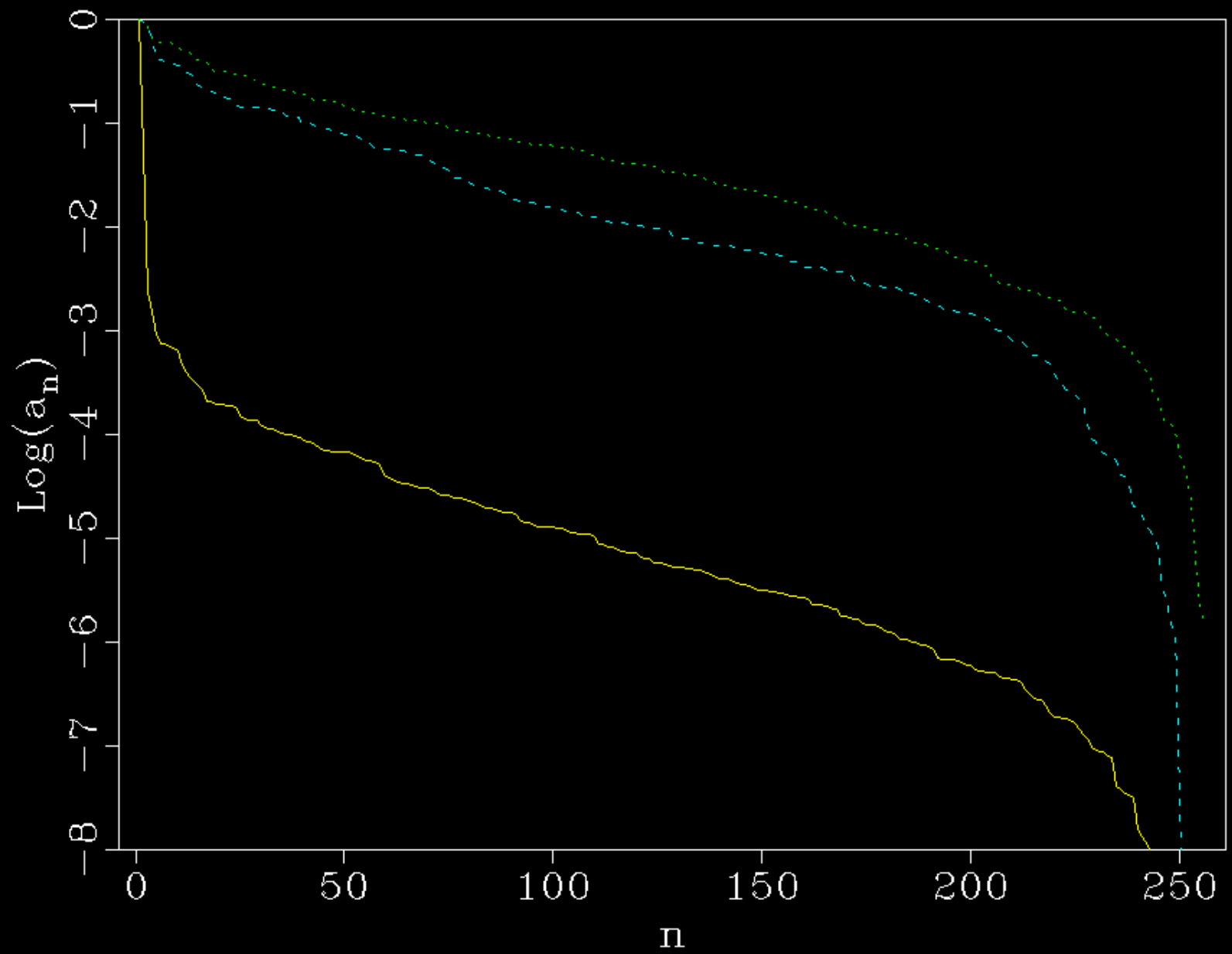


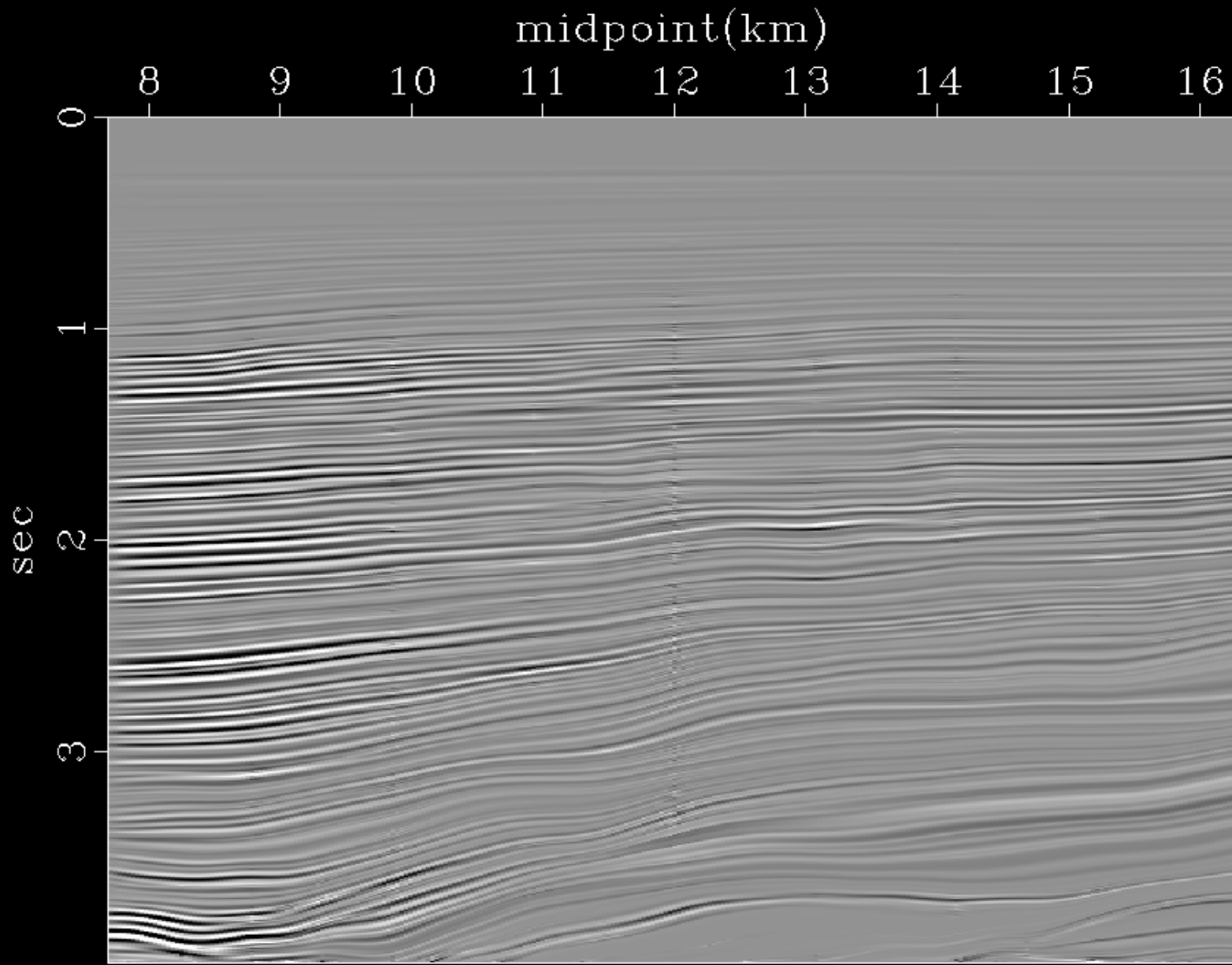


Seislet Transform

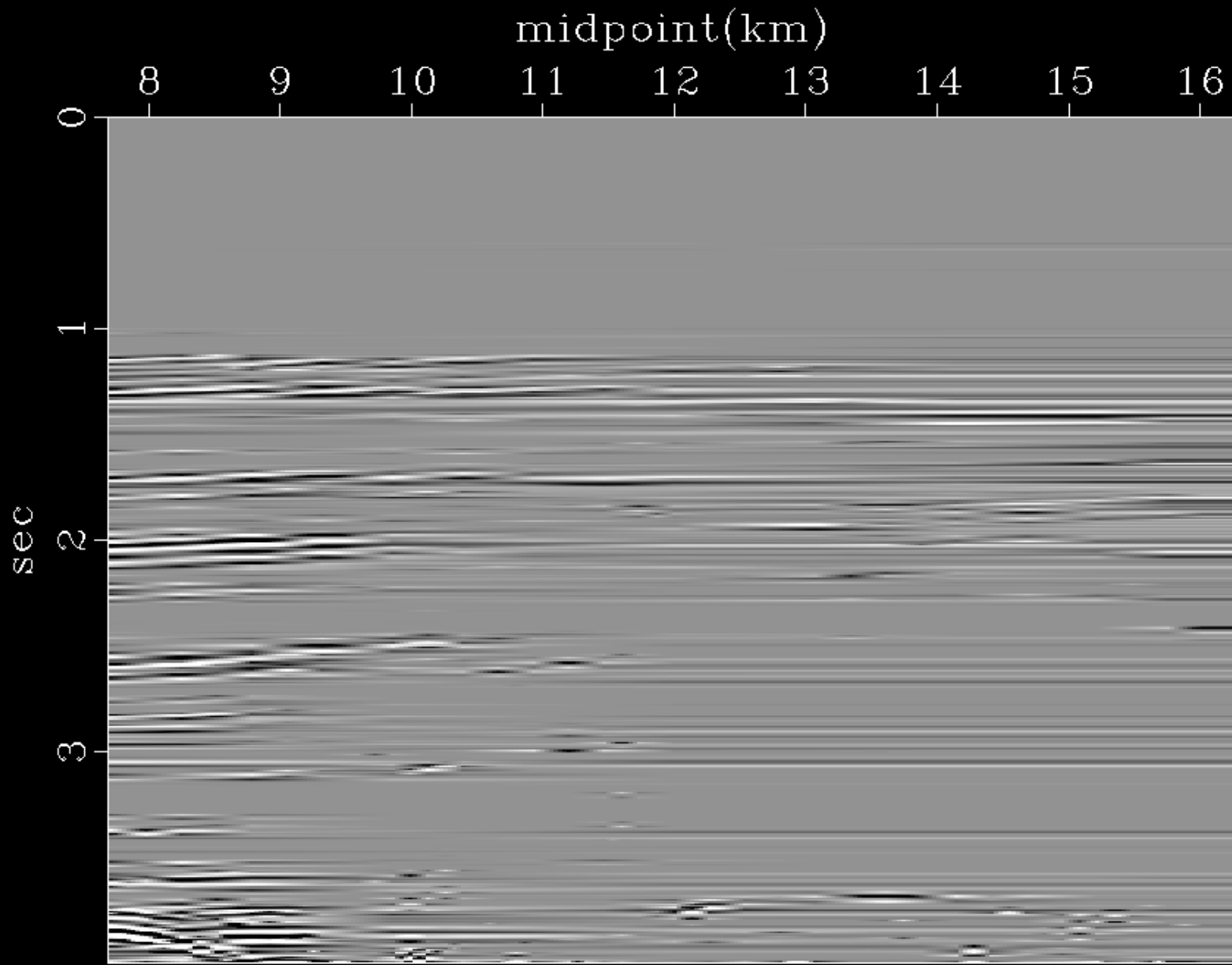


Wavelet Transform

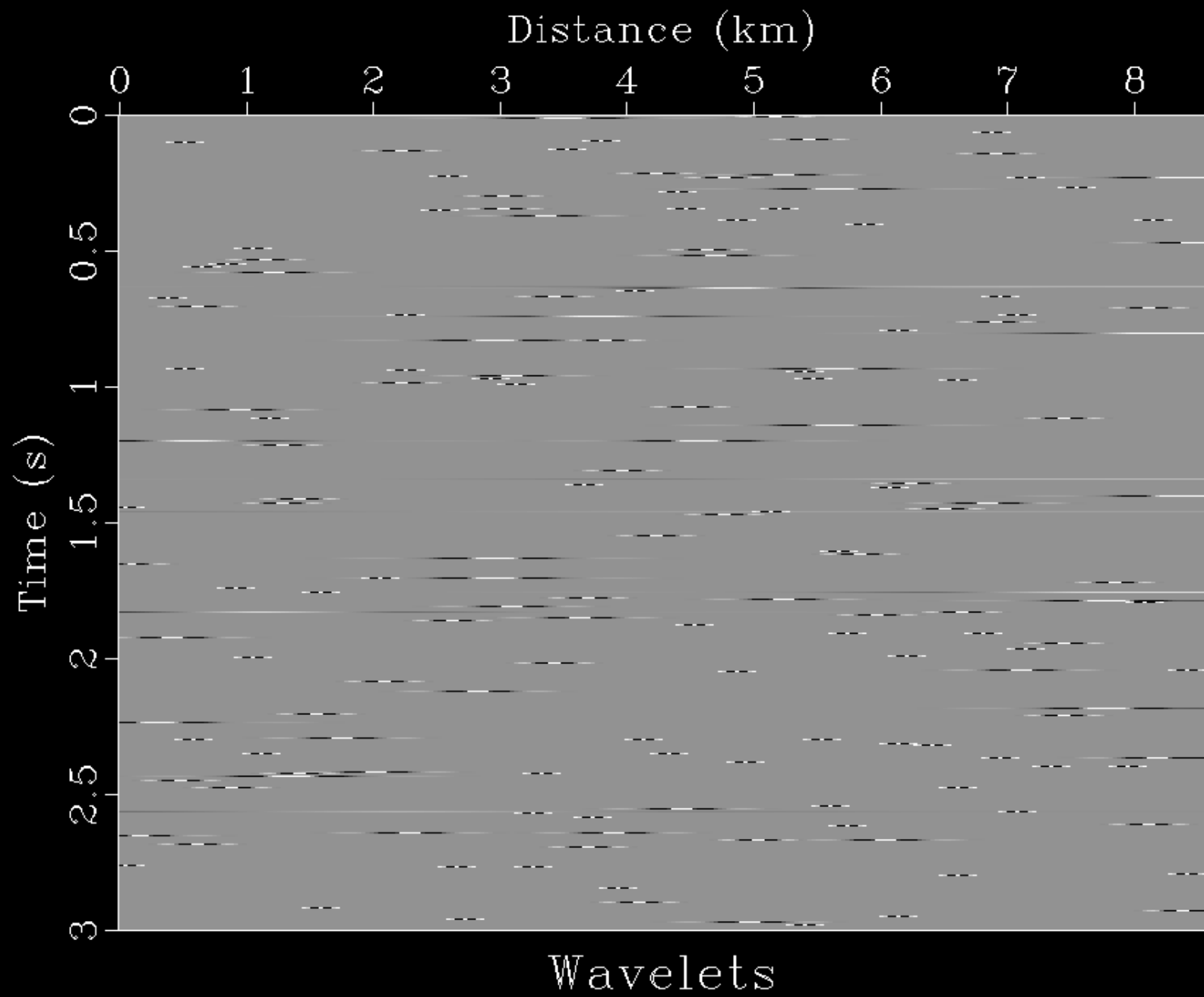


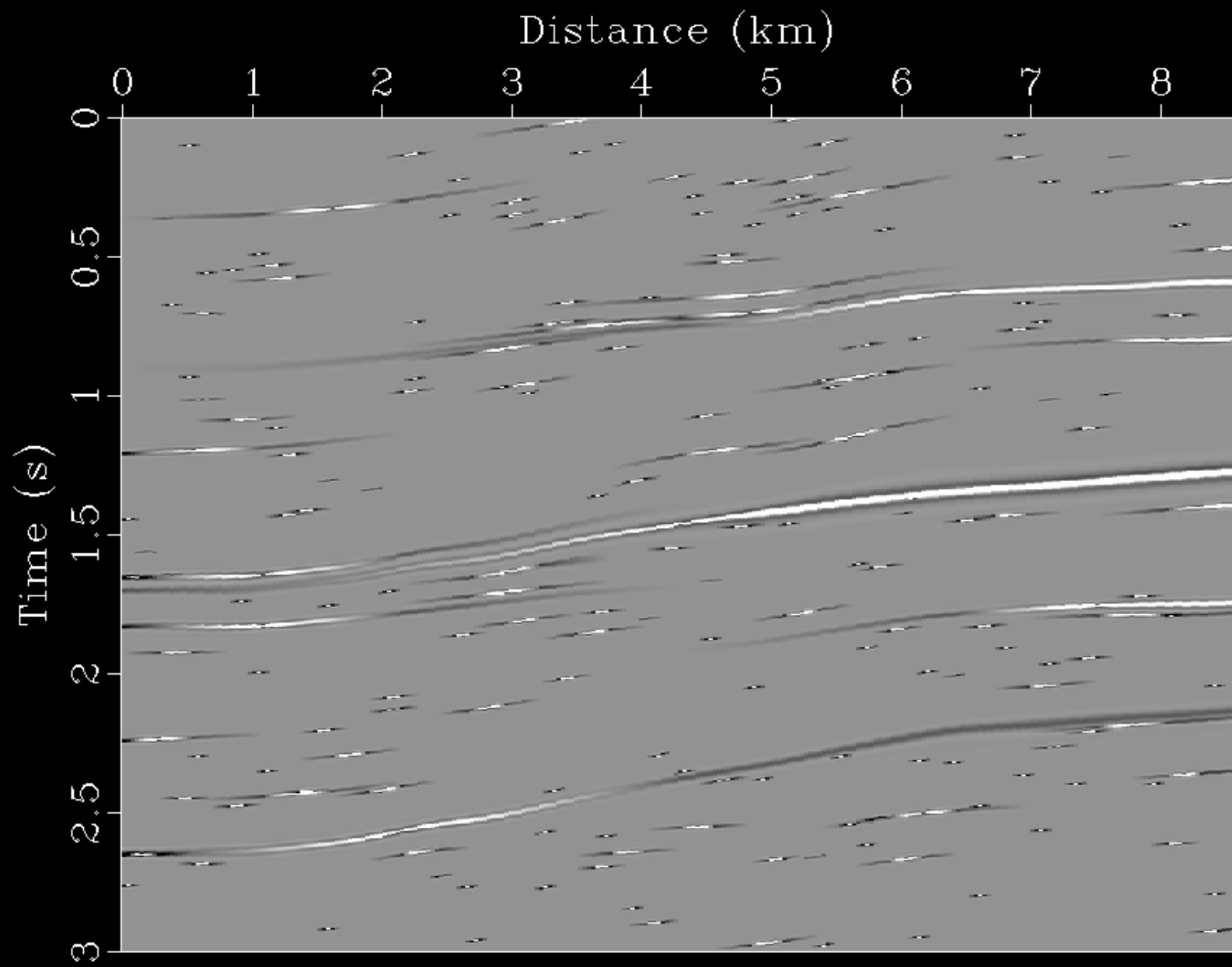


Inverse Seislet Transform (1%)



Inverse Wavelet Transform (1%)





Seislets

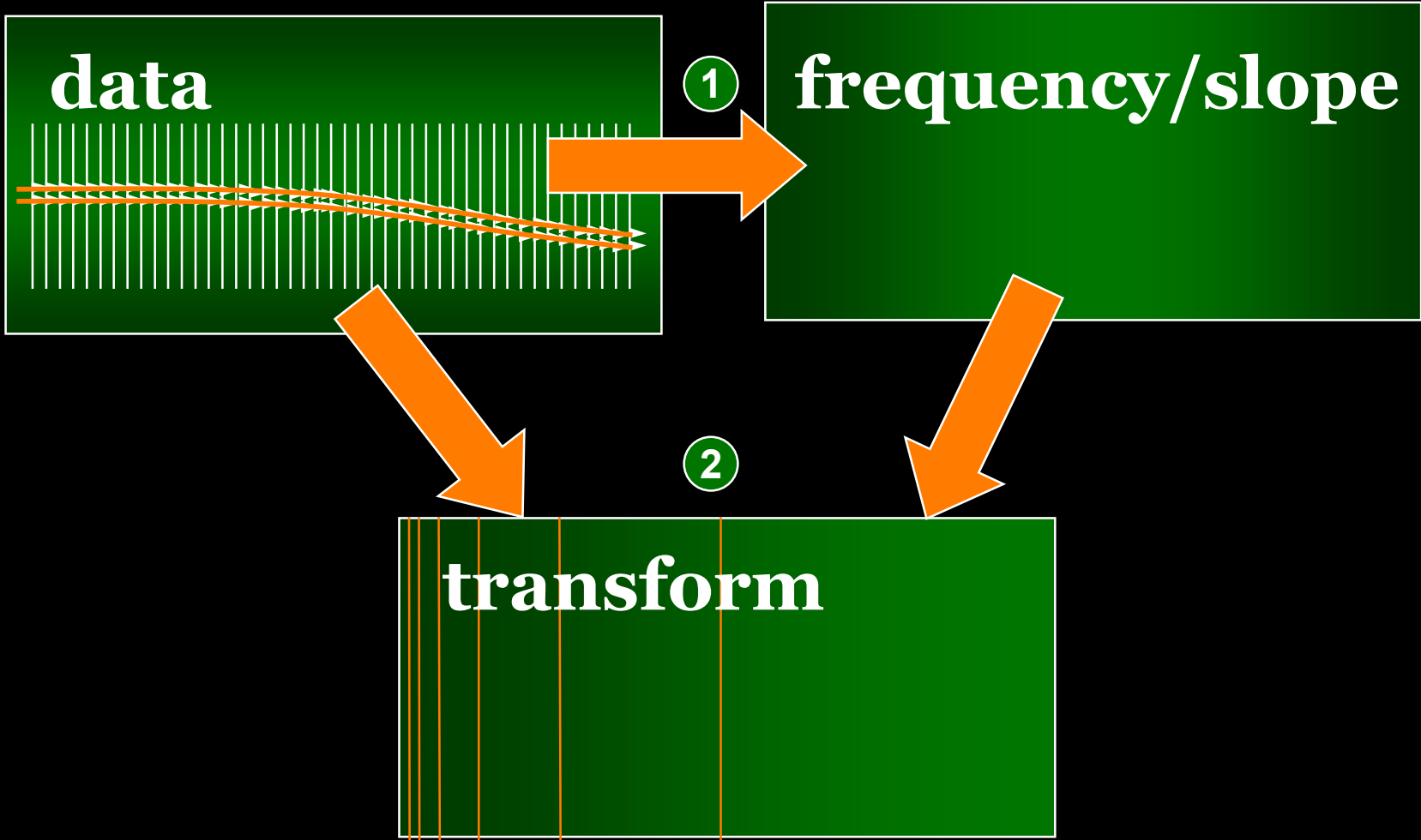


# Outline

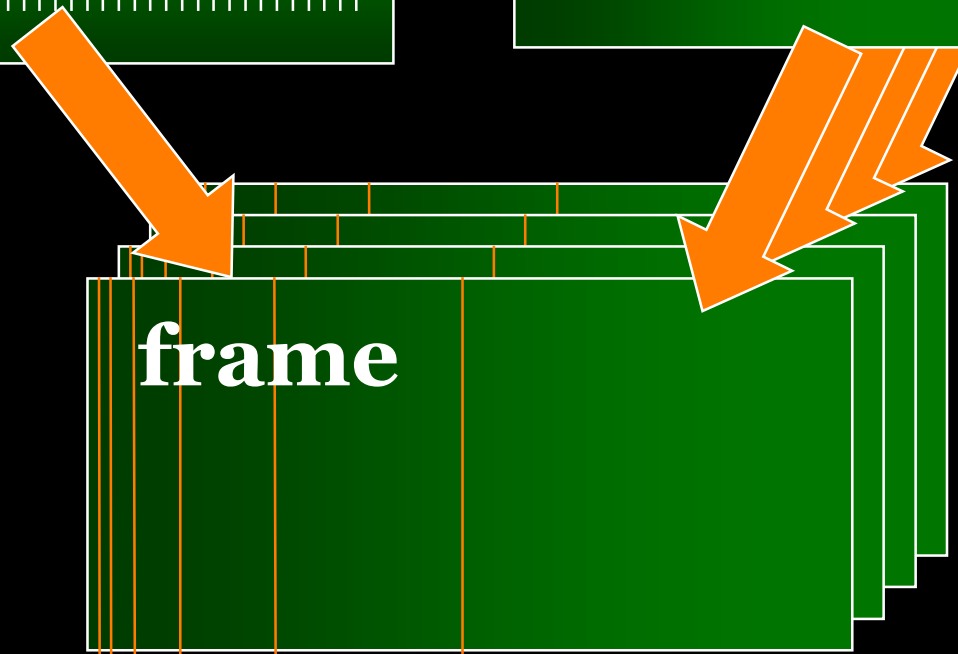
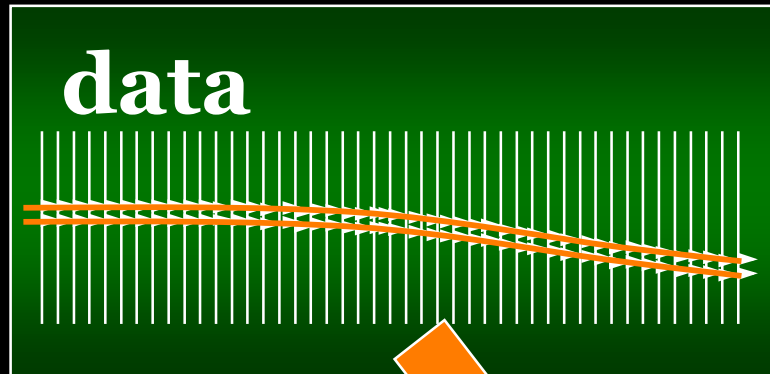


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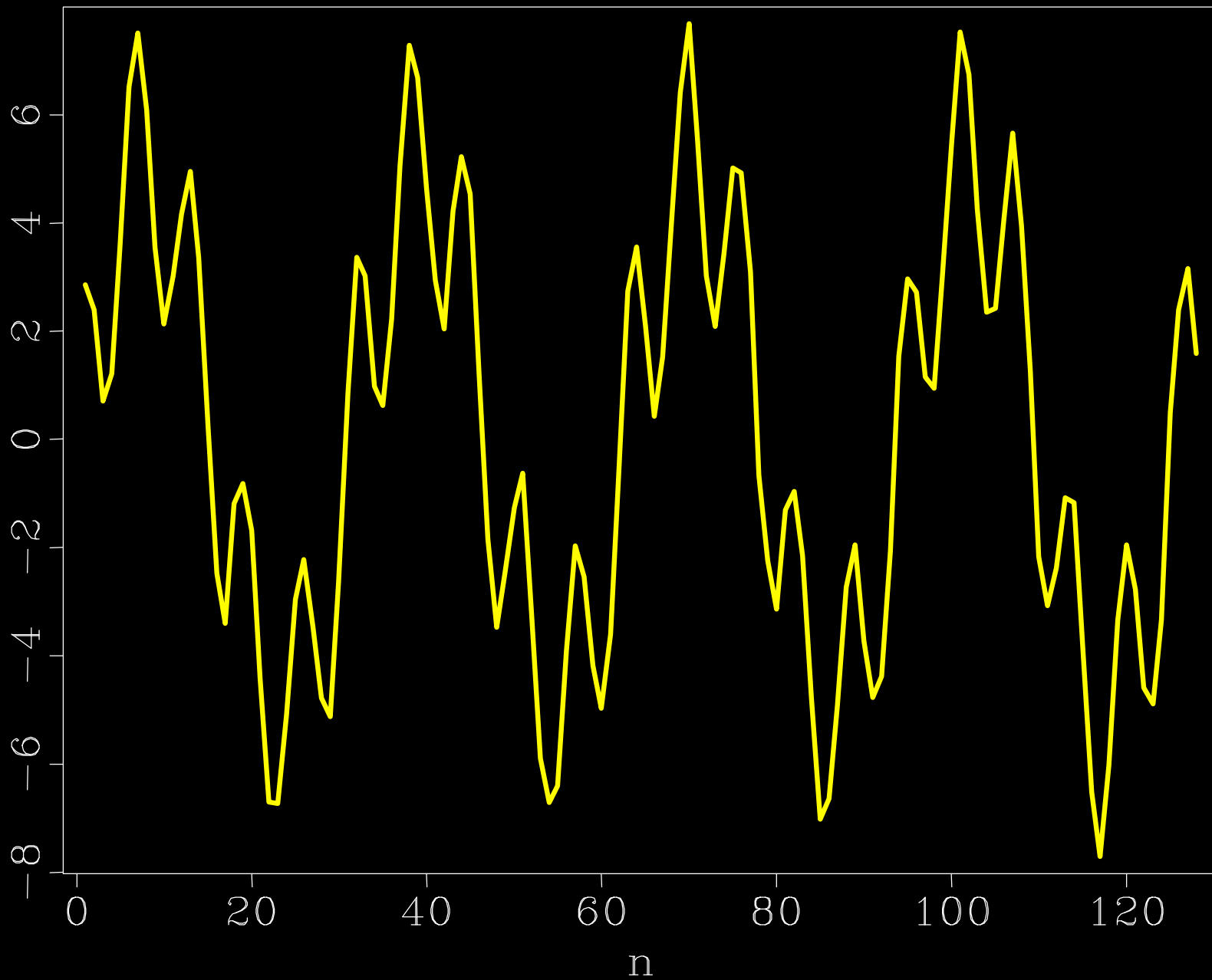
# Seislet Transform



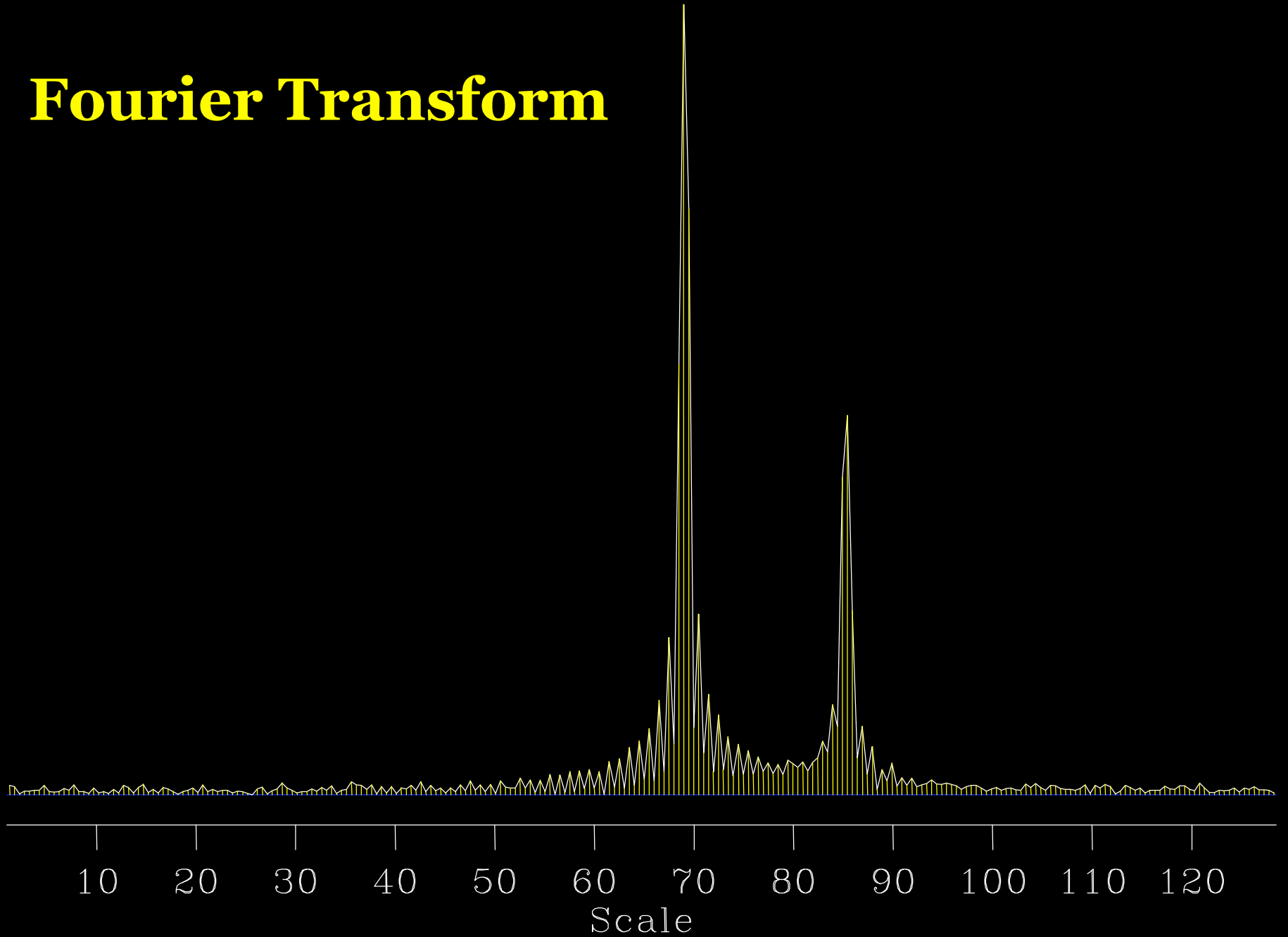
# Seislet Frame



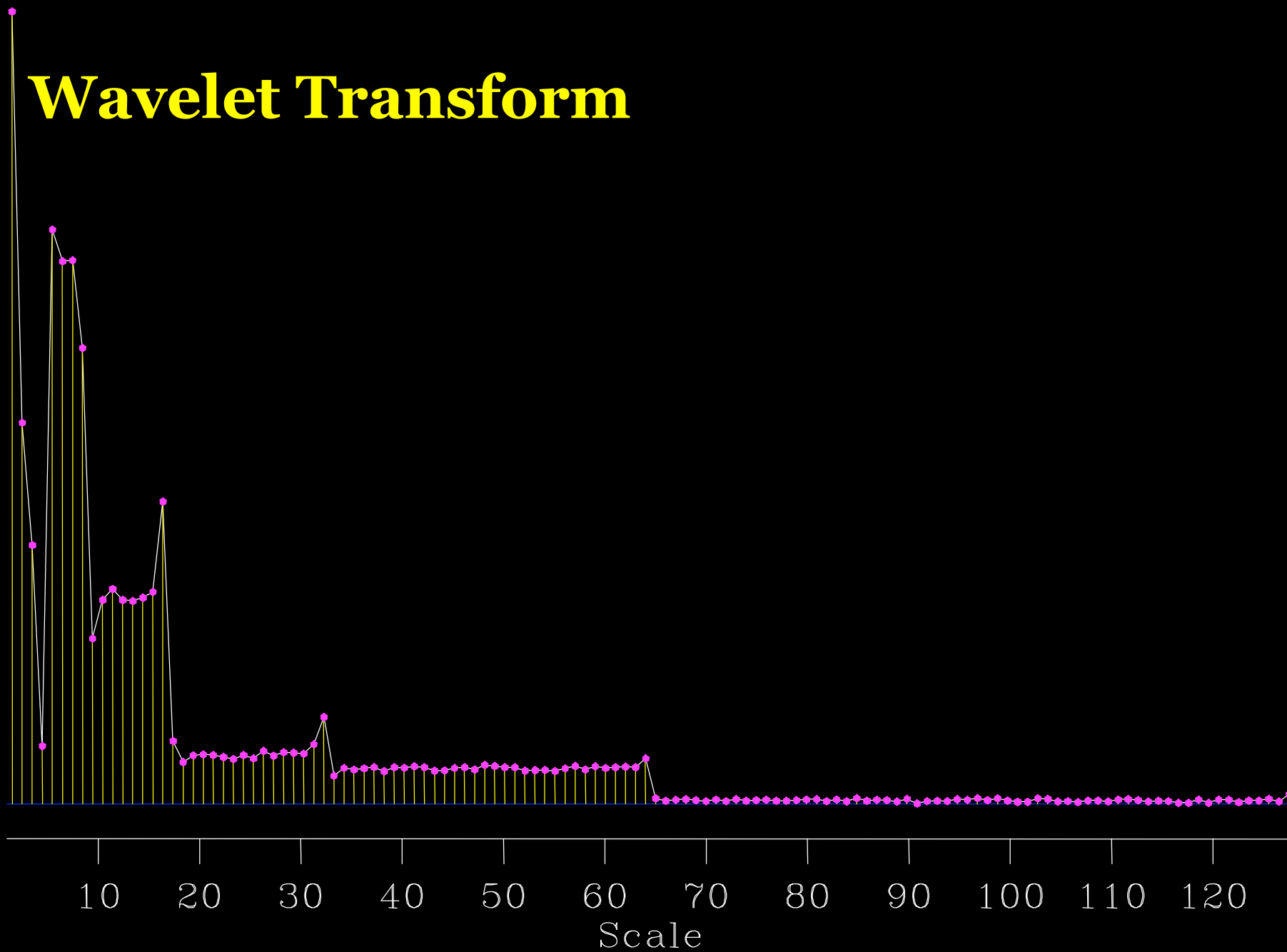
# Synthetic Mixed Sinusoidal Signal



# Fourier Transform



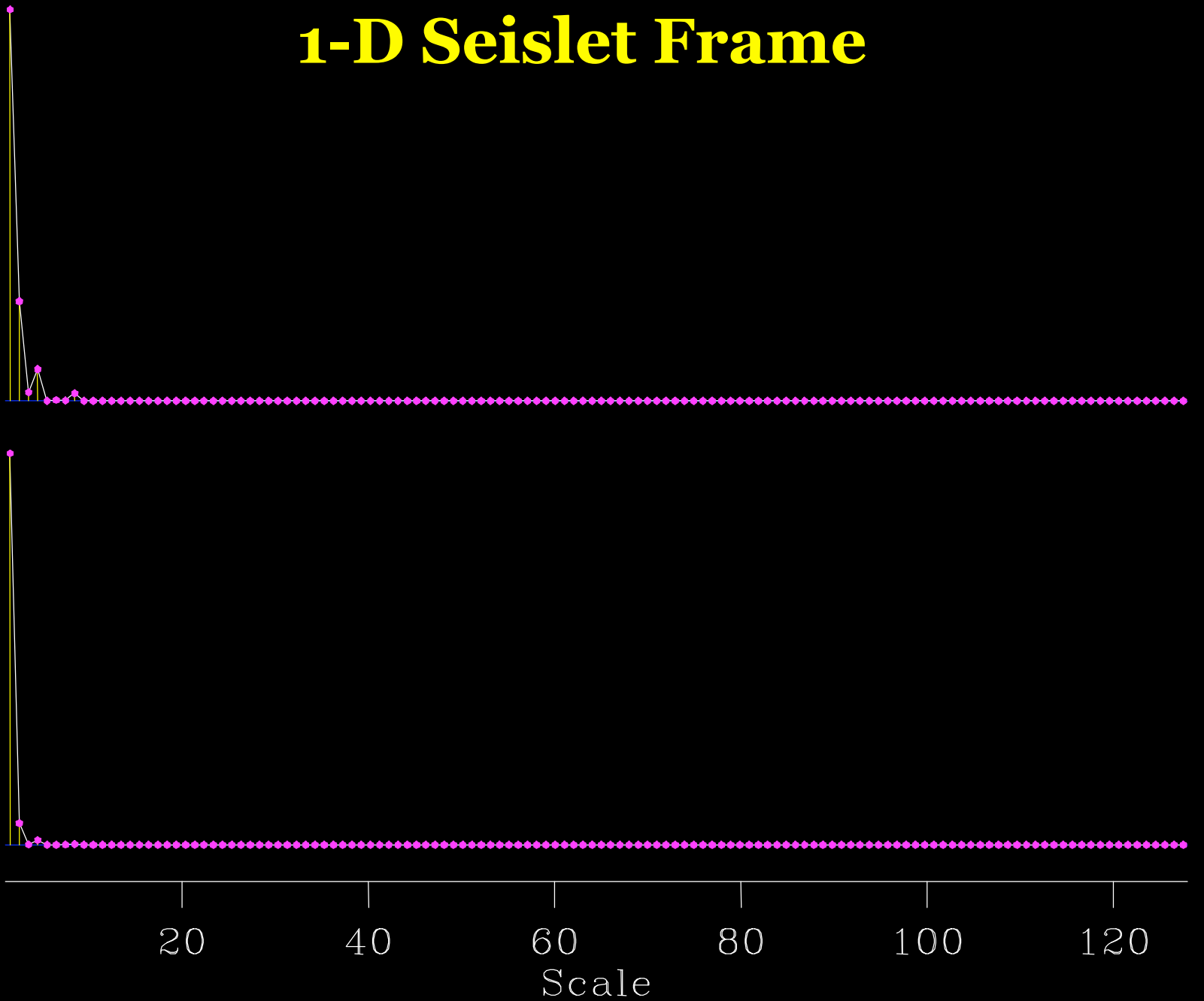
# Wavelet Transform



# 1-D Seislet Frame

First

Second

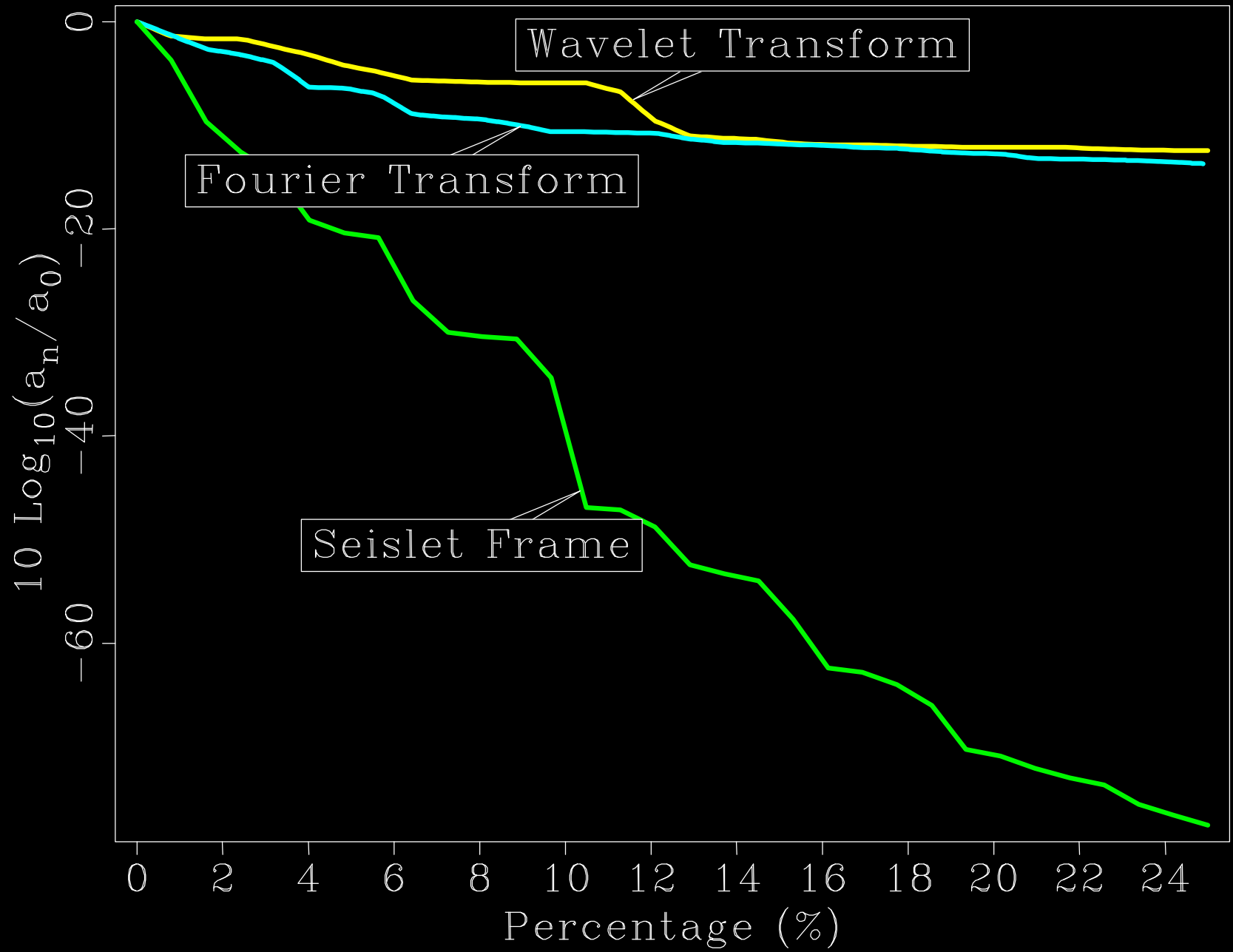


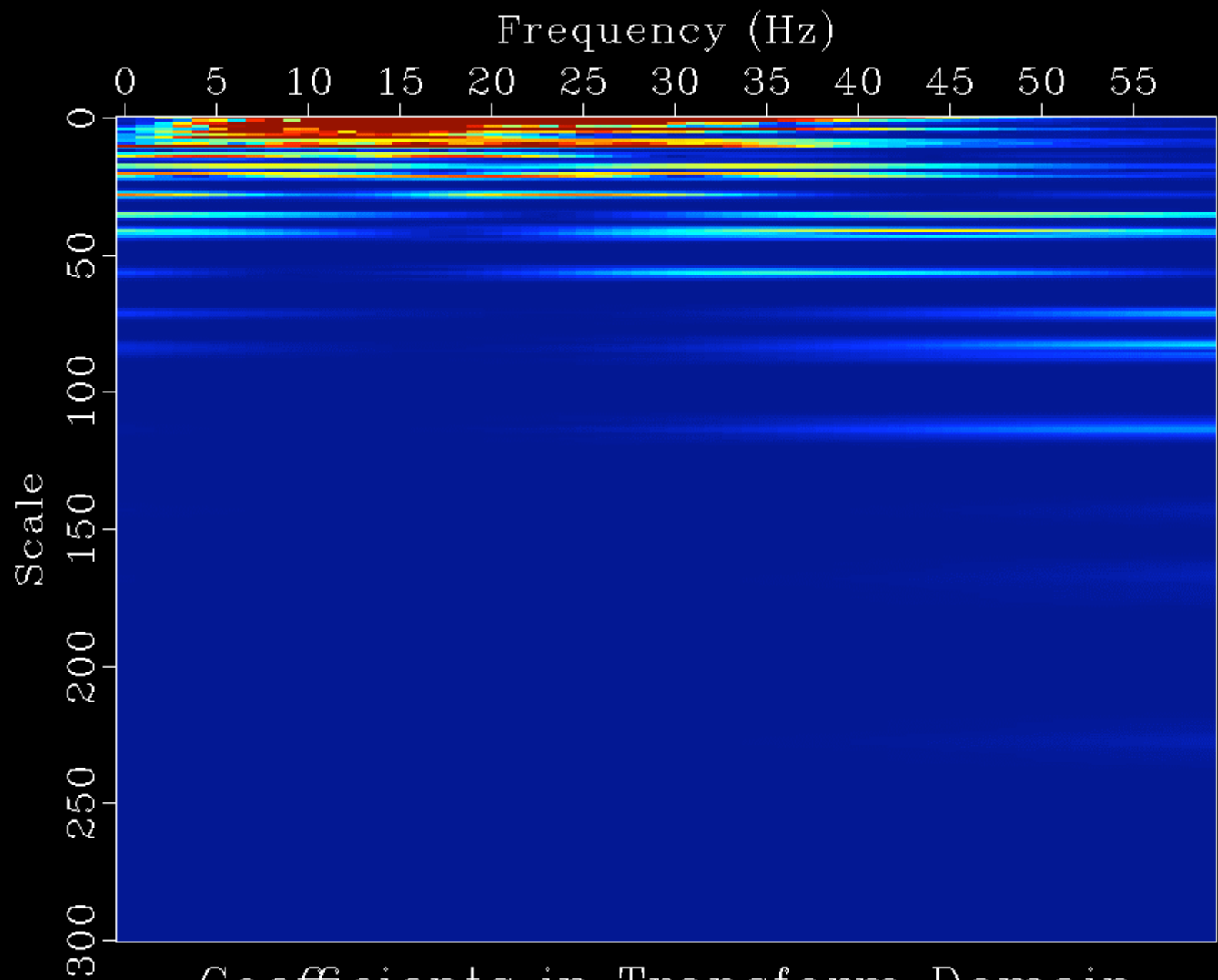
# Compressive Sensing

- ✓ (Donoho, 2006; Candes, Romberg, and Tao, 2006)
- ✓  $\min(\|d - K T t\|_2 + \|t\|_1)$

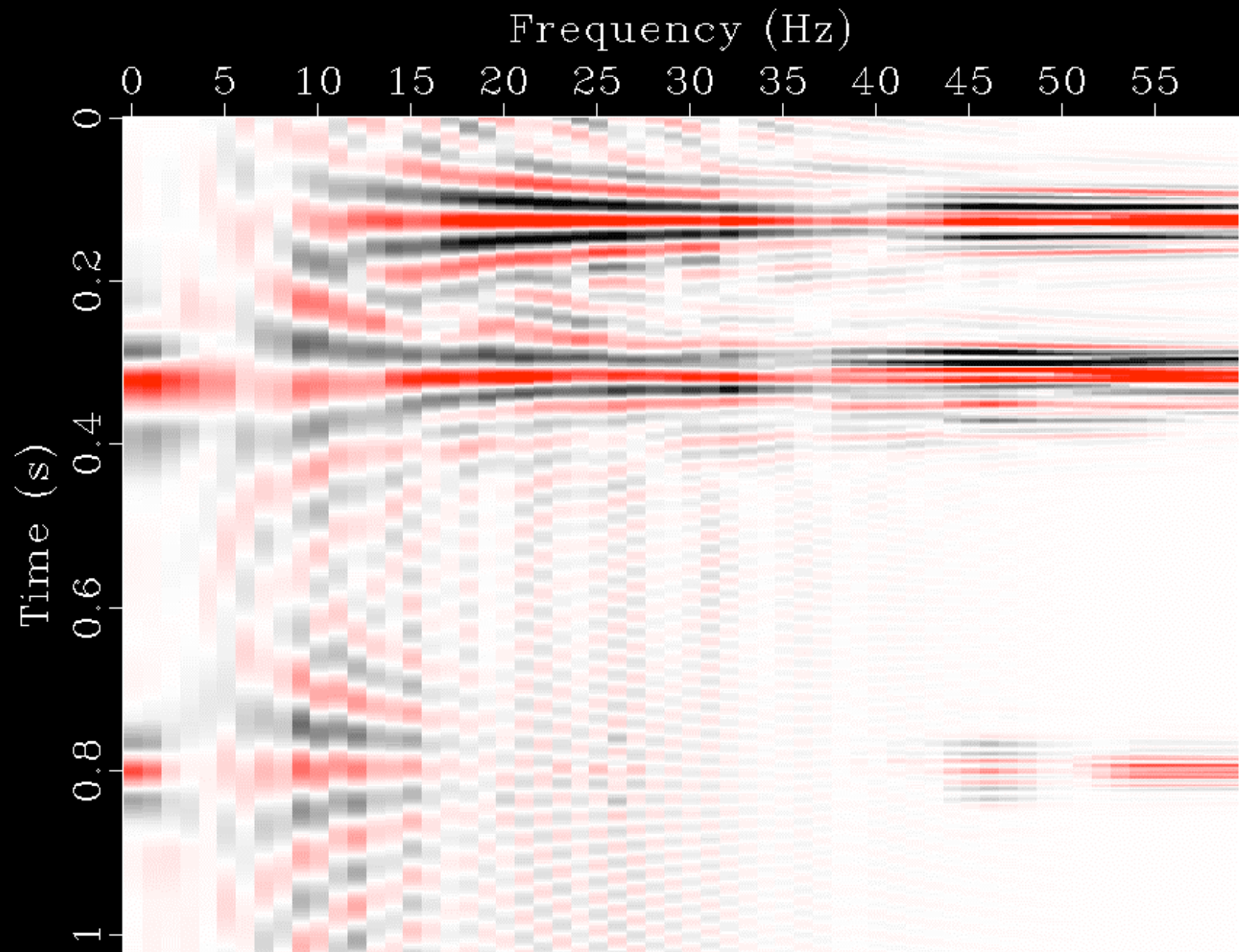


# Compression Ratio





Coefficients in Transform Domain



Time-frequency Spectra

# Conclusions



- ✓ **Seismic reflection imaging is an important practical application**
- ✓ **Seismic data exhibit predictable multiscale patterns**
- ✓ **Seismic patterns can be captured by seislet transform and frame**

# Publishing Numerical Experiments

- ✓ **Make them reproducible!**
- ✓ **All demonstrated computational results are**
  - ✓ **documented (reproducible)**
  - ✓ **open-source (reviewable)**
    - ✓ **MADAGASCAR software package**
    - ✓ **<http://rsf.sourceforge.net/>**
- ✓ **Special issue of CiSE Jan/Feb 2009**

