Introduction to Wavelets

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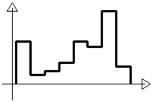
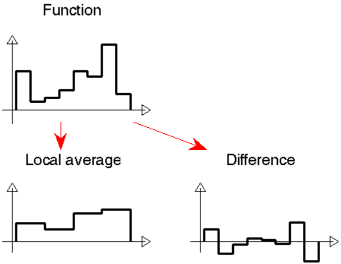
Wavelet multiresolution decomposition

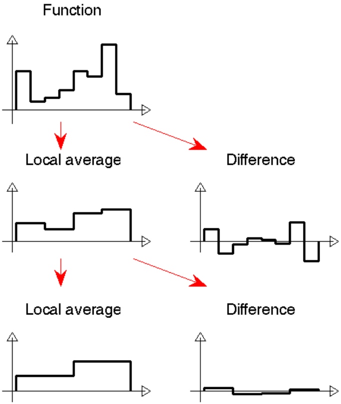
Function



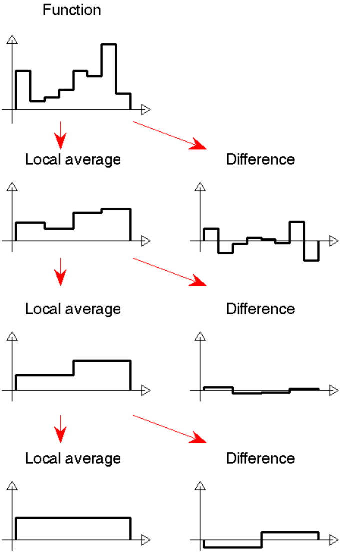
Wavelet multiresolution decomposition



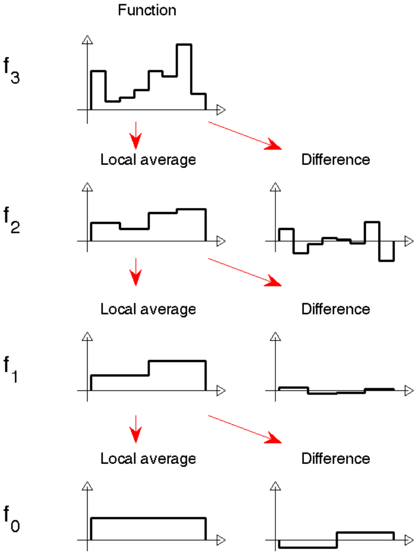
Wavelet multiresolution decomposition



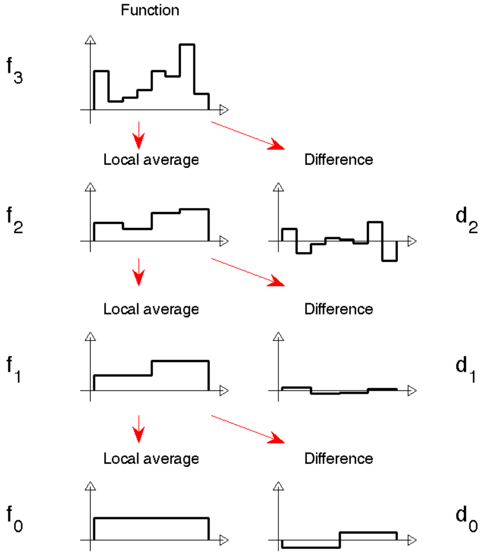
Wavelet multiresolution decomposition



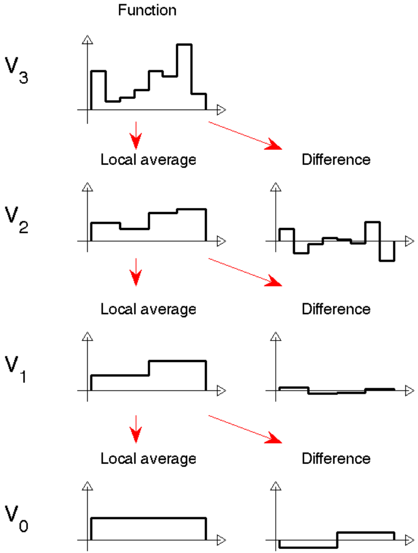
Wavelet multiresolution decomposition



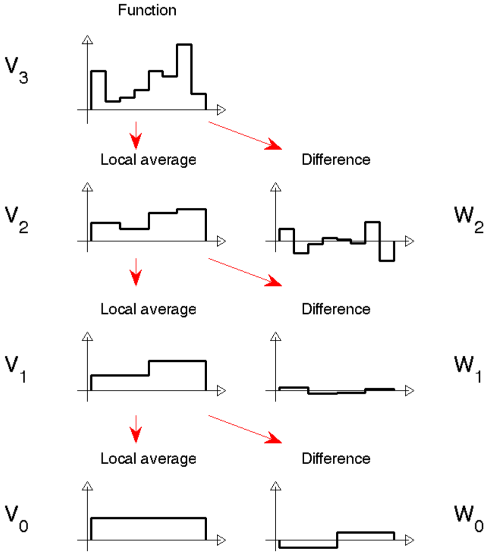
Wavelet multiresolution decomposition



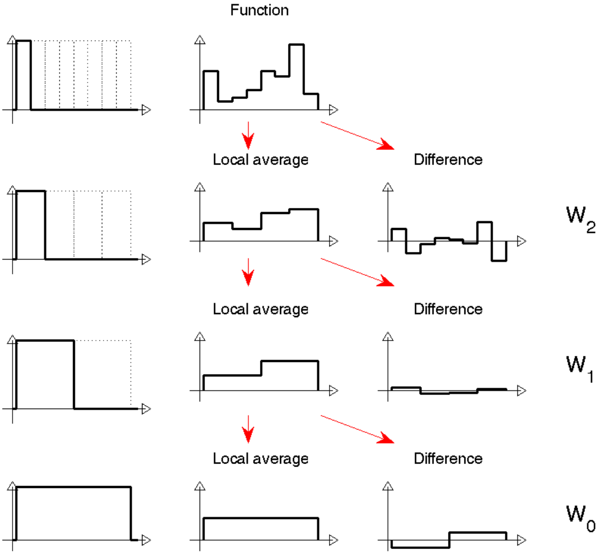
Wavelet multiresolution decomposition



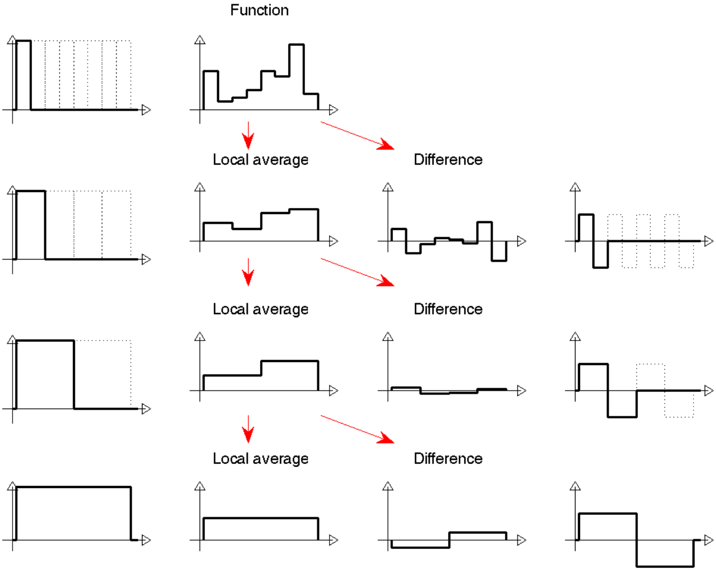
Wavelet multiresolution decomposition



Wavelet multiresolution decomposition



Wavelet multiresolution decomposition



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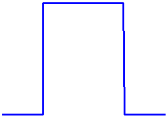
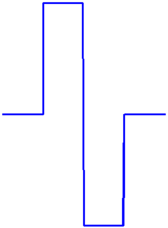


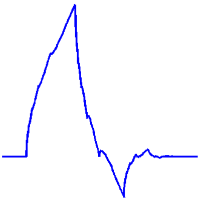
Scaling function $\phi(x)$



Mother wavelet $\psi(x)$

Wavelet examples

Daubechies 4



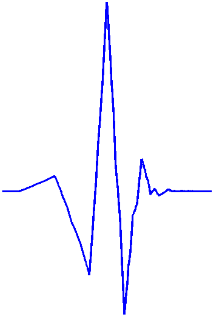
Scaling function $\phi(x)$

Mother wavelet $\psi(x)$

Wavelet examples

Daubechies 6

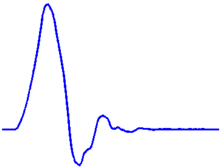
Scaling function $\phi(x)$



Mother wavelet $\psi(x)$

Wavelet examples

Daubechies 8



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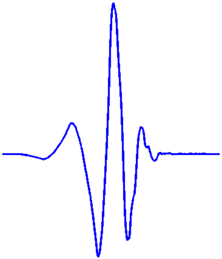
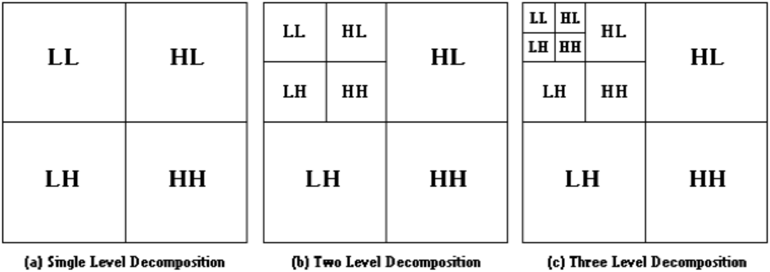


Mother wavelet $\psi(x)$

Wavelet based image compression

- Wavelets successful in image compression. Eg: JPEG 2000 standard (Daubechies (9,7) biorthogonal wavelets), FBI fingerprint database, . . .
- Consider image as a function and use 2D wavelets.



- Compress by thresholding + coding + quantization.

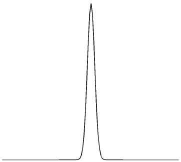
Wavelet based image compression

Time Frequency Representation of Signals

Given a discrete signal $f(n)$, $n = 0, 1, \ldots$.

Time representation — total localization in time

$$f(n) = \sum_k f(k)\delta(n-k)$$



Basis functions

Time Frequency Representation of Signals

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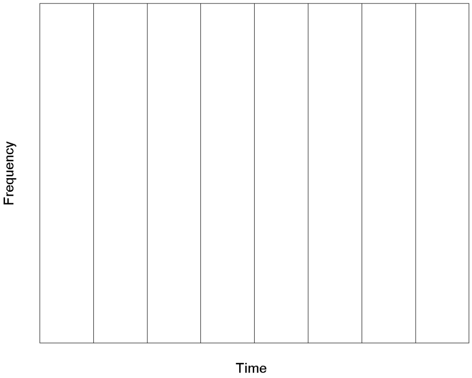
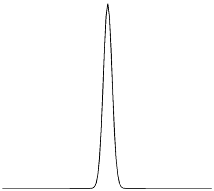
Time representation — total localization in time





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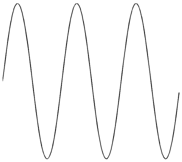
Time Frequency Representation of Signals

Given a discrete signal $f(n)$, $n = 0, 1, \ldots$.

Fourier representation — total localization in frequency

$$f(n) = \sum_j \hat{f}(j) exp(inj2\pi/N)$$



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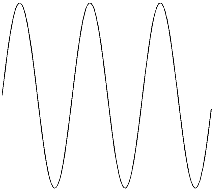
Fourier representation — total localization in frequency



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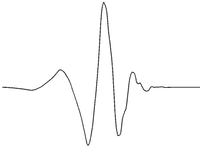
Time Frequency Representation of Signals

Given a discrete signal $f(n)$, $n = 0, 1, \ldots$.

Wavelet representation — localization in time and frequency

$$f(n) = \sum_{j,k} w_{j,k} \psi_{j,k}(n/2\pi)$$



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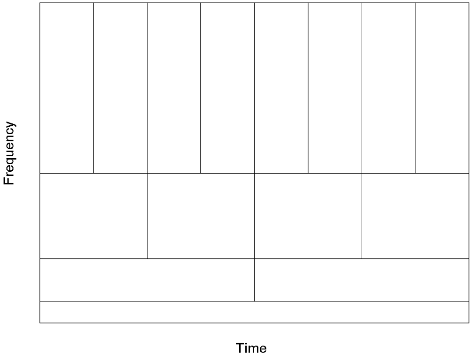

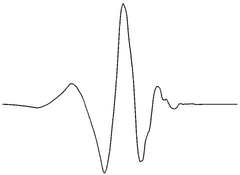

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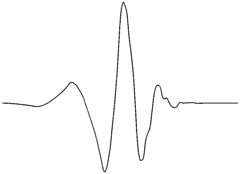
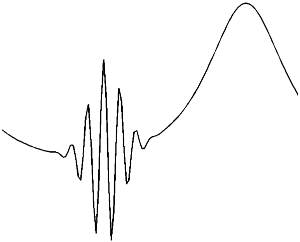
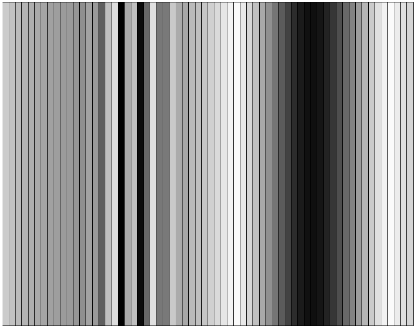


$$f(n) = \sum_{j,k} w_{j,k} \psi_{j,k}(n/2\pi)$$

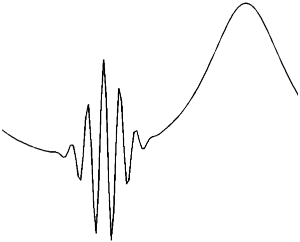
Time Frequency Representation of Signals
Example

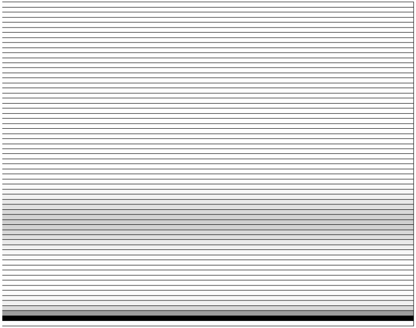
Time representation — total localization in time



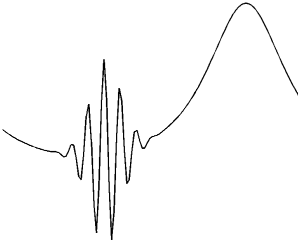


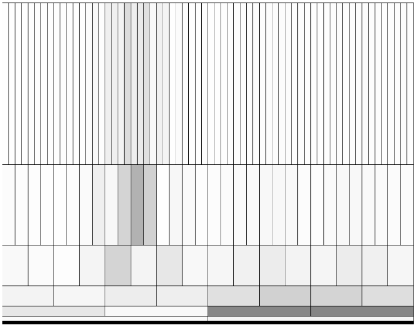
Fourier representation — total localization in frequency





Wavelet representation — localization in time and frequency





Wavelets – some contributors

- Strömberg – first continuous wavelet
- Morlet, Grossman – *"wavelet"*
- Meyer, Mallat, Coifman – multiresolution analysis
- Daubechies – compactly supported wavelets
- Beylkin, Cohen, Dahmen, DeVore – PDE methods using wavelets
- Sweldens – lifting, second generation wavelets

. . . and many, many more.