Find an equation for the tangent to the graph of \( f(x) \) at the point \( P(2, f(2)) \) when
\[
f(x) = \frac{5}{1 - 3x}.
\]

1. \( y + \frac{3}{5}x + \frac{2}{5} = 0 \)
2. \( y = \frac{3}{5}x - \frac{11}{5} \) correct
3. \( y = \frac{1}{5}x - \frac{9}{5} \)
4. \( y = 3x - 7 \)
5. \( y + \frac{1}{5}x + \frac{3}{5} = 0 \)

**Explanation:**
If \( x = 2 \), then \( f(2) = -1 \), so we have to find an equation for the tangent line to the graph of
\[
f(x) = \frac{5}{1 - 3x}
\]
at the point \( (2, -1) \). Now the Newtonian quotient for \( f \) at a general point \( (x, f(x)) \) is given by
\[
\frac{f(x + h) - f(x)}{h}.
\]
First let’s compute the numerator of the Newtonian Quotient:
\[
f(x + h) - f(x) = \frac{5}{1 - 3(x + h)} - \frac{5}{1 - 3x}
\]
\[
= \frac{5(1 - 3x) - 5(1 - 3(x + h))}{(1 - 3x)(1 - 3(x + h))}
\]
\[
= \frac{15h}{(1 - 3h)(1 - 3(x + h))}.
\]
Thus
\[
\frac{f(x + h) - f(x)}{h} = \frac{15}{(1 - 3x)(1 - 3(x + h))}.
\]

Hence
\[
f'(x) = \lim_{h \to 0} \frac{15}{(1 - 3(x + h))(1 - 3x)} = \frac{15}{(1 - 3x)^2}.
\]
At \( x = 2 \), therefore,
\[
f'(2) = \frac{15}{(1 - 6)^2} = \frac{3}{5},
\]
so by the point slope formula an equation for the tangent line at \( (2, -1) \) is
\[
y + 1 = \frac{3}{5}(x - 2)
\]
which after simplification becomes
\[
y = \frac{3}{5}x - \frac{11}{5}.
\]

Find \( \frac{dy}{dx} \) when
\[
\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 5.
\]

1. \( \frac{dy}{dx} = \frac{3}{2}(x^{-3/2}) \)
2. \( \frac{dy}{dx} = -\frac{2}{3}(y^{-3/2}) \) correct
3. \( \frac{dy}{dx} = \frac{2}{3}(xy)^{1/2} \)
4. \( \frac{dy}{dx} = \frac{3}{2}(xy)^{1/2} \)
5. \( \frac{dy}{dx} = \frac{2}{3}(y^{3/2}) \)
6. \( \frac{dy}{dx} = -\frac{3}{2}(x^{3/2}) \)

**Explanation:**
Differentiating implicitly with respect to $x$, we see that
\[-\frac{1}{2} \left( \frac{2}{x\sqrt{x}} + \frac{3}{y\sqrt{y}} \frac{dy}{dx} \right) = 0.\]
Consequently,
\[
\frac{dy}{dx} = -\frac{2}{3} \left( \frac{y}{x} \right)^{3/2}.
\]

003 10.0 points

Determine the value of $\frac{dy}{dt}$ at $x = 3$ when
\[ y = x^2 - 3x \]
and $\frac{dx}{dt} = 3$.

1. $\frac{dy}{dt} \bigr|_{x=3} = 5$
2. $\frac{dy}{dt} \bigr|_{x=3} = 9$ correct
3. $\frac{dy}{dt} \bigr|_{x=3} = 3$
4. $\frac{dy}{dt} \bigr|_{x=3} = 1$
5. $\frac{dy}{dt} \bigr|_{x=3} = 7$

Explanation:
Differentiating implicitly with respect to $t$ we see that
\[
\frac{dy}{dt} = (2x - 3) \frac{dx}{dt} = 3(2x - 3).
\]
At $x = 3$, therefore,
\[
\frac{dy}{dt} = 3(3) = 9.
\]

004 10.0 points

Determine $f'(x)$ when
\[ f(x) = \frac{\sin(x) - 1}{\sin(x) + 4}. \]

1. $f'(x) = -\frac{3\cos(x)}{(\sin(x) + 4)^2}$
2. $f'(x) = \frac{5\sin(x)\cos(x)}{\sin(x) + 4}$
3. $f'(x) = -\frac{5\cos(x)}{(\sin(x) + 4)^2}$
4. $f'(x) = -\frac{3\sin(x)\cos(x)}{\sin(x) + 4}$
5. $f'(x) = \frac{5\cos(x)}{(\sin(x) + 4)^2}$ correct
6. $f'(x) = \frac{3\cos(x)}{\sin(x) + 4}$

Explanation:
By the Quotient Rule,
\[
f'(x) = \frac{(\sin(x) + 4)\cos(x) - (\sin(x) - 1)\cos(x)}{(\sin(x) + 4)^2}.
\]
But
\[
(\sin(x) + 4)\cos(x) - (\sin(x) - 1)\cos(x) = 5\cos(x).
\]
Thus
\[
f'(x) = \frac{5\cos(x)}{(\sin(x) + 4)^2}.
\]

keywords: derivative of trig functions, derivative, quotient rule

005 10.0 points

The points $P$ and $Q$ on the graph of
\[ y^2 - xy - 5y + 10 = 0 \]
have the same $x$-coordinate $x = 2$. Find the point of intersection of the tangent lines to the graph at $P$ and $Q$.

1. intersect at $= \left( \frac{5}{7}, \frac{20}{7} \right)$ correct
2. intersect at $= \left( \frac{10}{3}, \frac{20}{7} \right)$
3. intersect at $= \left( \frac{5}{7}, -\frac{20}{7} \right)$
4. intersect at $= \left(\frac{20}{7}, \frac{5}{3}\right)$

5. intersect at $= \left(\frac{5}{3}, \frac{20}{7}\right)$

**Explanation:**

The respective $y$-coordinates at $P$, $Q$ are the solutions of

\[
y^2 - xy - 5y + 10 = 0
\]

at $x = 2$; i.e., the solutions of

\[
y^2 - 7y + 10 = (y - 5)(y - 2) = 0.
\]

Thus

\[
P = (2, 5), \quad Q = (2, 2).
\]

To determine the tangent lines we need also the value of the derivative at $P$ and $Q$. But by implicit differentiation,

\[
2y \frac{dy}{dx} - (x + 5) \frac{dy}{dx} - y = 0.
\]

so

\[
\frac{dy}{dx} = \frac{y}{2y - x - 5}.
\]

Thus

\[
\frac{dy}{dx} \bigg|_P = \frac{5}{3}, \quad \frac{dy}{dx} \bigg|_Q = -\frac{2}{3}.
\]

By the point-slope formula, therefore, the equation of the tangent line at $P$ is

\[
y - 5 = \frac{5}{3}(x - 2),
\]

while that at $Q$ is

\[
y - 2 = -\frac{2}{3}(x - 2).
\]

Consequently, the tangent lines at $P$ and $Q$ are

\[
y = \frac{5}{3}x + \frac{5}{3}
\]

and

\[
y + \frac{2}{3}x = \frac{10}{3}
\]

respectively. These two tangent lines intersect at $= \left(\frac{5}{7}, \frac{20}{7}\right)$.

---

**006 10.0 points**

At noon, ship $A$ is 150 miles due west of ship $B$. Ship $A$ is sailing south at 30 mph while ship $B$ is sailing north at 10 mph.

At what speed is the distance between the ships changing at 5:00 pm?

1. speed = 34 mph
2. speed = 38 mph
3. speed = 32 mph **correct**
4. speed = 30 mph
5. speed = 36 mph

**Explanation:**

Let $a = a(t)$ be the distance travelled by ship $A$ up to time $t$ and $b = b(t)$ the distance travelled by ship $B$. Then, if $s = s(t)$ is the distance between the ships, the relative positions and directions of movement of the ships are shown in
By Pythagoras, therefore,
\[ s^2 = (a + b)^2 + (150)^2. \]

Thus, after implicit differentiation,
\[ 2s \frac{ds}{dt} = 2(a + b) \left( \frac{da}{dt} + \frac{db}{dt} \right), \]
in which case,
\[ \frac{ds}{dt} = (30 + 10) \left( \frac{a + b}{s} \right) = 40 \left( \frac{a + b}{s} \right) \]
when ships A and B are travelling at respective speeds 30 mph and 10 mph.

But at 5:00 pm,
\[ a = 150, \quad b = 50, \quad s = 250 \]

Consequently, at 5:00 pm, the distance between the ships is increasing at a speed \[ \frac{ds}{dt} \big|_{t=5} = 32 \text{ mph}. \]

**007 10.0 points**

Determine \( \frac{dy}{dx} \) when
\[ y \cos(x^2) = 5. \]

1. \( \frac{dy}{dx} = 2xy \cos(x^2) \)
2. \( \frac{dy}{dx} = 2xy \tan(x^2) \) correct
3. \( \frac{dy}{dx} = -2xy \tan(x^2) \)
4. \( \frac{dy}{dx} = 2xy \cot(x^2) \)
5. \( \frac{dy}{dx} = -2xy \cot(x^2) \)
6. \( \frac{dy}{dx} = -2xy \sin(x^2) \)

**Explanation:**

We have to express \( dy/dt \) in terms of \( x, y \) and \( dx/dt \). But by Pythagoras’ theorem,
\[ x^2 + y^2 = 25, \]

After implicit differentiation with respect to \( x \) we see that
\[ -2xy \sin(x^2) + y' \cos(x^2) = 0. \]

Consequently,
\[ \frac{dy}{dx} = \frac{2xy \sin(x^2)}{\cos(x^2)} = 2xy \tan(x^2). \]

**008 10.0 points**

A 5 foot ladder is leaning against a wall. If the foot of the ladder is sliding away from the wall at a rate of 12 ft/sec, at what speed is the top of the ladder falling when the foot of the ladder is 4 feet away from the base of the wall?

1. speed = 16 ft/sec correct
2. speed = 15 ft/sec
3. speed = \( \frac{49}{3} \) ft/sec
4. speed = \( \frac{46}{3} \) ft/sec
5. speed = \( \frac{47}{3} \) ft/sec

**Explanation:**

Let \( y \) be the height of the ladder when the foot of the ladder is \( x \) feet from the base of the wall as shown in figure.

We have\[ x^2 + y^2 = 25, \]
so by implicit differentiation,
\[ 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0. \]

In this case
\[ \frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}. \]

But again by Pythagoras, if \( x = 4 \), then \( y = 3 \). Thus, if the foot of the ladder is moving away from the wall at a speed of
\[ \frac{dx}{dt} = 12 \text{ ft/sec}, \]
and \( x = 4 \), then the velocity of the top of the ladder is given by
\[ \frac{dy}{dt} = -\frac{4}{3} \frac{dx}{dt}. \]

Consequently, the speed at which the top of the ladder is falling is
\[ \text{speed} = \left| \frac{dy}{dt} \right| = 16 \text{ ft/sec}. \]

The slope, \( m \), of the tangent line at the point \( P(2, f(2)) \) on the graph of \( f \) is the value of the derivative
\[ f'(x) = 2x + 3 \]
at \( x = 2 \), i.e., \( m = 7 \). On the other hand, \( f(2) = 12 \).

Thus by the point-slope formula, an equation for the tangent line at \( P(2, f(2)) \) is
\[ y - 12 = 7(x - 2), \]
which after simplification becomes
\[ y = 7x - 2. \]

Consequently, the tangent line at \( P \) has
\[ y\text{-intercept} = -2 \] .

10 10.0 points

Determine the derivative of
\[ f(x) = 2 \arcsin \left( \frac{x}{3} \right). \]

1. \( f'(x) = \frac{6}{\sqrt{9 - x^2}} \)
2. \( f'(x) = \frac{3}{\sqrt{9 - x^2}} \)
3. \( f'(x) = \frac{2}{\sqrt{1 - x^2}} \)
4. \( f'(x) = \frac{6}{\sqrt{1 - x^2}} \)
5. \( f'(x) = \frac{2}{\sqrt{9 - x^2}} \) correct
6. \( f'(x) = \frac{3}{\sqrt{1 - x^2}} \)

Explanation:
Use of
\[ \frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1 - x^2}}, \]
together with the Chain Rule shows that
\[ f'(x) = \frac{2}{\sqrt{1 - (x/3)^2}} \left( \frac{1}{3} \right). \]
Consequently,
\[
f'(x) = \frac{2}{\sqrt{9 - x^2}}.
\]

011 10.0 points

Find the derivative of
\[f(x) = \sin^{-1}(e^{3x}).\]

1. \(f'(x) = \frac{3e^{3x}}{\sqrt{1 - e^{6x}}} \text{ correct}\)

2. \(f'(x) = \frac{3}{\sqrt{1 - e^{6x}}}\)

3. \(f'(x) = \frac{3e^{3x}}{1 + e^{6x}}\)

4. \(f'(x) = \frac{3}{1 + e^{6x}}\)

5. \(f'(x) = \frac{e^{3x}}{1 + e^{6x}}\)

6. \(f'(x) = \frac{1}{1 + e^{6x}}\)

7. \(f'(x) = \frac{1}{\sqrt{1 - e^{6x}}}\)

8. \(f'(x) = \frac{e^{3x}}{\sqrt{1 - e^{6x}}}\)

Explanation:
Since
\[
d x \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}}, \quad d e^{ax} = ae^{ax},
\]
the Chain Rule ensures that
\[
f'(x) = \frac{3e^{3x}}{\sqrt{1 - e^{6x}}}.
\]

012 10.0 points

The height of a triangle is increasing at a rate of 4 cm/min while its area is increasing at a rate of 3 sq. cms/min.

At what speed is the base of the triangle changing when the height of the triangle is 3 cms and its area is 15 sq. cms?

1. speed = \(\frac{32}{3}\) cms/min

2. speed = 11 cms/min

3. speed = 12 cms/min

4. speed = \(\frac{35}{3}\) cms/min

5. speed = \(\frac{34}{3}\) cms/min \text{ correct}

Explanation:
Let \(b\) be the length of the base and \(h\) the height of the triangle. Then the triangle has
\[\text{area} = A = \frac{1}{2}bh.\]

Thus by the Product Rule,
\[
\frac{dA}{dt} = \frac{1}{2} \left( b \frac{dh}{dt} + h \frac{db}{dt} \right),
\]
and so
\[
\frac{db}{dt} = \frac{1}{h} \left( 2 \frac{dA}{dt} - b \frac{dh}{dt} \right) = \frac{2}{h} \left( \frac{dA}{dt} - \frac{A}{h} \frac{dh}{dt} \right),
\]

since \(b = 2A/h\). Thus, when
\[
\frac{dh}{dt} = 4, \quad \text{and} \quad \frac{dA}{dt} = 3,
\]
we see that
\[
\frac{db}{dt} = \frac{2}{h} \left( 3 - \frac{4A}{h} \right) \text{ cms/min}.
\]

Consequently, at the moment when \(h = 3\) and \(A = 15\),

the base length is changing at a
\[
\text{speed} = \frac{34}{3} \text{ cms/min}
\]
Use linear approximation with \( a = 16 \) to estimate the number \( \sqrt{16.2} \) as a fraction.

1. \( \sqrt{16.2} \approx 4 \frac{1}{40} \) correct
2. \( \sqrt{16.2} \approx 4 \frac{1}{80} \)
3. \( \sqrt{16.2} \approx 4 \frac{3}{80} \)
4. \( \sqrt{16.2} \approx 4 \frac{1}{20} \)
5. \( \sqrt{16.2} \approx 4 \frac{1}{16} \)

Explanation: For a general function \( f \), its linear approximation at \( x = a \) is defined by

\[
L(x) = f(a) + f'(a)(x-a)
\]

and for values of \( x \) near \( a \)

\[
f(x) \approx L(x) = f(a) + f'(a)(x-a)
\]

provides a reasonable approximation for \( f(x) \).

Now set

\[
f(x) = \sqrt{x}, \quad f'(x) = \frac{1}{2\sqrt{x}}.
\]

Then, if we can calculate \( \sqrt{a} \) easily, the linear approximation

\[
\sqrt{a+h} \approx \sqrt{a} + \frac{h}{2\sqrt{a}}
\]

provides a very simple method via calculus for computing a good estimate of the value of \( \sqrt{a+h} \) for small values of \( h \).

In the given example we can thus set

\[
a = 16, \quad h = \frac{2}{10}.
\]

For then

\[
\sqrt{16.2} \approx 4 \frac{1}{40}.
\]
B. If $F_2(x) = f(x)g(x)$, then
$$F'_2(x) = f'(x)g(x) + f(x)g'(x).$$

C. If $F_3(x) = f(x)/g(x)$, then
$$F'_3(x) = \frac{f'(x)g(x) + f(x)g'(x)}{g(x)^2}.$$ 

Which of these statements are true?

1. C only
2. B and C only
3. A and B only
4. A only
5. all of them
6. B only correct
7. none of them
8. A and C only

**Explanation:**

By the respective Product and Quotient Rules,
$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$
while
$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}.$$ 

Applying these to
$$F(x) = \left( f(x) - \frac{1}{f(x)} \right)^2$$
we see that
$$F'(x) = 2f'(x) \left( f(x) - \frac{1}{f(x)^2} \right),$$

Consequently,

A. Not True
B. True
C. Not True

---

**016 10.0 points**

Find the derivative of $f$ when
$$f(x) = \sqrt{x}(2x + 7).$$

1. $f'(x) = \frac{4x + 7}{x\sqrt{x}}$
2. $f'(x) = \frac{6x + 7}{2\sqrt{x}}$ correct
3. $f'(x) = \frac{6x - 7}{2\sqrt{x}}$
4. $f'(x) = \frac{4x - 7}{x\sqrt{x}}$
5. $f'(x) = \frac{6x - 7}{x\sqrt{x}}$
6. $f'(x) = \frac{4x + 7}{2\sqrt{x}}$

**Explanation:**

By the Product Rule
$$f'(x) = \frac{2x + 7}{2\sqrt{x}} + 2\sqrt{x}.$$ 

After simplification this becomes
$$f'(x) = \frac{2x + 7 + 4x}{2\sqrt{x}} = \frac{6x + 7}{2\sqrt{x}}.$$

---

**017 10.0 points**

Find the derivative of
$$f(x) = (\sin^{-1}(3x))^2.$$ 

1. $f'(x) = \frac{3}{\sqrt{1 - 9x^2}} \sin^{-1}(3x)$
2. $f'(x) = \cos(3x) \sin(3x)$

3. $f'(x) = \frac{6}{\sqrt{9-x^2}} \sin^{-1}(3x)$

4. $f'(x) = 6\cos(3x) \sin(3x)$

5. $f'(x) = \frac{3}{\sqrt{9-x^2}} \sin^{-1}(3x)$

6. $f'(x) = \frac{6}{\sqrt{1-9x^2}} \sin^{-1}(3x)$ correct

**Explanation:**

The Chain Rule together with

$$\frac{d}{dx} (\sin^{-1}(ax)) = \frac{a}{\sqrt{1-a^2x^2}}$$

shows that

$$f'(x) = \frac{6}{\sqrt{1-9x^2}} \sin^{-1}(3x).$$

Thus the slope of the tangent line at $P$ is

$$f'\left(\frac{\pi}{4}\right) = 9\sec^2\left(\frac{\pi}{4}\right) = 18.$$

By the point-slope formula, therefore, an equation for the tangent line at $P$ is given by

$$y - 9 = 18\left(x - \frac{\pi}{4}\right),$$

which after simplification becomes

$$y = 18x + 9\left(1 - \frac{\pi}{2}\right).$$

**018 10.0 points**

Find an equation for the tangent line to the graph of $f$ at the point $P\left(\frac{\pi}{4}, f\left(\frac{\pi}{4}\right)\right)$ when $f(x) = 9\tan(x)$.

1. $y = 10x + 2\left(1 - \frac{\pi}{4}\right)$

2. $y = 18x + 9\left(1 - \frac{\pi}{2}\right)$ correct

3. $y = 13x + 18\left(1 - \frac{\pi}{4}\right)$

4. $y = 14x + 3\left(1 - \frac{\pi}{4}\right)$

5. $y = 17x + 14\left(1 - \frac{\pi}{2}\right)$

**Explanation:**

When $x = \frac{\pi}{4}$, then $f(x) = 9$, so $P = \left(\frac{\pi}{4}, 9\right)$. Now

$$f'(x) = 9\sec^2(x).$$

We see that

$$f'(\frac{\pi}{4}) = 9\sec^2\left(\frac{\pi}{4}\right) = 18.$$

Consequently,

$$f'(x) = \frac{1}{(5x-1)^2}.$$
A circle of radius \( r \) has area \( A \) and circumference \( C \) are given respectively by

\[
A = \pi r^2, \quad C = 2\pi r.
\]

If \( r \) varies with time \( t \), for what value of \( r \) is the rate of change of \( A \) with respect to \( t \) twice the rate of change of \( C \) with respect to \( t \)?

1. \( r = \frac{1}{2} \)
2. \( r = 2\pi \)
3. \( r = 2 \) correct
4. \( r = 1 \)
5. \( r = \frac{\pi}{2} \)
6. \( r = \pi \)

**Explanation:**
Differentiating

\[
A = \pi r^2, \quad C = 2\pi r
\]

implicitly with respect to \( t \) we see that

\[
\frac{dA}{dt} = 2\pi r \frac{dr}{dt}, \quad \frac{dC}{dt} = 2\pi \frac{dr}{dt}.
\]

Thus the rate of change, \( dA/dt \), of area is twice the rate of change, \( dC/dt \), of circumference when

\[
\frac{dA}{dt} = 2 \frac{dC}{dt},
\]

i.e., when

\[
2\pi r \frac{dr}{dt} = 2 \left(2\pi \frac{dr}{dt}\right).
\]

This happens when

\[
\boxed{r = 2}.
\]

**022 10.0 points**

Determine \( f'(x) \) when

\[
f(x) = \tan^{-1}\left(\frac{x}{\sqrt{3-x^2}}\right).
\]

(Hint: first simplify \( f(x) \).)

1. \( f'(x) = \frac{\sqrt{3}}{\sqrt{3+x^2}} \)
2. \( f'(x) = \frac{x}{\sqrt{x^2-3}} \)

**Explanation:**
By the Quotient Rule,

\[
f'(x) = \frac{(\sin x) \sin x - (\cos x)(2 - \cos x)}{\sin^2 x}
\]

\[
= \frac{(\cos^2 x + \sin^2 x) - 2 \cos x}{\sin^2 x}.
\]

But

\[
\cos^2 x + \sin^2 x = 1.
\]

Consequently,

\[
\boxed{f'(x) = \frac{1 - 2 \cos x}{\sin^2 x}}.
\]

keywords: DerivTrig, DerivTrigExam, quotient rule

**021 10.0 points**

Find the derivative of

\[
f(x) = \frac{2 - \cos x}{\sin x}.
\]

1. \( f'(x) = \frac{1 - 2 \cos x}{\sin^2 x} \) correct
2. \( f'(x) = \frac{2 + \sin x}{\cos^2 x} \)
3. \( f'(x) = \frac{1 + 2 \cos x}{\sin^2 x} \)
4. \( f'(x) = \frac{2 - \sin x}{\cos x} \)
5. \( f'(x) = \frac{1 + 2 \cos x}{\sin x} \)
6. \( f'(x) = \frac{2 - \sin x}{\cos^2 x} \)
3. \( f'(x) = \frac{1}{\sqrt{3 - x^2}} \) correct

4. \( f'(x) = \frac{\sqrt{3}}{\sqrt{3 - x^2}} \)

5. \( f'(x) = \frac{x}{x^2 + 3} \)

Explanation:
If \( \tan \theta = \frac{x}{\sqrt{3 - x^2}} \),
then by Pythagoras' theorem applied to the right triangle

\[ \sqrt{3} \quad \theta \]
\[ \sqrt{3 - x^2} \quad x \]

we see that
\[ \sin \theta = \frac{x}{\sqrt{3}}. \]

Thus
\[ f(x) = \sin^{-1}\left( \frac{x}{\sqrt{3}} \right). \]
Consequently,
\[ f'(x) = \frac{1}{\sqrt{3 - x^2}}. \]

Alternatively, we can differentiate \( f \) using the Chain Rule and the fact that
\[ \frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}. \]


2. \( f'(x) = -\frac{\sec^2(x)}{(8 + 3 \sec(x) \tan(x))^2} \)

3. \( f'(x) = \frac{8 \sec(x) + 3}{(8 + 3 \sec(x))^2} \)

4. \( f'(x) = \frac{8 \cos(x) + 3}{(8 \cos(x) + 3)^2} \)

5. \( f'(x) = \frac{8 + 3 \cos(x)}{(8 \cos(x) + 3)^2} \) correct

6. \( f'(x) = \frac{8 \sin(x) + 3 \cos(x)}{(8 \cos(x) + 3)^2} \)

Explanation:
Now,
\[ \frac{\tan(x)}{8 + 3 \sec(x)} = \frac{\sin(x)}{8 + \frac{3}{\cos(x)}} = \frac{\sin(x)}{8 \cos(x) + 3}. \]

Thus
\[ f'(x) = \frac{\cos(x)}{8 \cos(x) + 3} + \frac{8 \sin^2(x)}{(8 \cos(x) + 3)^2} \]
\[ = \frac{\cos(x)(8 \cos(x) + 3) + 8 \sin^2(x)}{(8 \cos(x) + 3)^2}. \]
Consequently,
\[ f'(x) = \frac{8 + 3 \cos(x)}{(8 \cos(x) + 3)^2}, \]

since \( \cos^2(x) + \sin^2(x) = 1 \).


024 10.0 points

If \( y = y(x) \) is defined implicitly by
\[ 6e^{5y} = 3xy + 6x, \]
find the value of \( dy/dx \) at \((x, y) = (1, 0)\).

1. \( \frac{dy}{dx} = -\frac{2}{9} \)
2. \( \frac{dy}{dx} = \frac{2}{9} \) correct
3. \( \frac{dy}{dx} = \frac{5}{27} \)
4. \( \frac{dy}{dx} = -\frac{7}{27} \)

5. \( \frac{dy}{dx} = -\frac{2}{11} \)

6. \( \frac{dy}{dx} = \frac{2}{11} \)

**Explanation:**
Differentiating

\[ 6e^{5y} = 3xy + 6x \]

implicitly with respect to \( x \) we see that

\[ 30e^{5y} \frac{dy}{dx} = 3y + 3x \frac{dy}{dx} + 6. \]

In this case,

\[ \frac{dy}{dx} \left( 30e^{5y} - 3x \right) = 3y + 6. \]

Consequently, at \( (x, y) = (1, 0) \) we see that

\[ \frac{dy}{dx} = \frac{2}{9} \]

**keywords:** implicit differentiation, exponential function,

**025 10.0 points**

Boyle’s Law states that when a sample of gas is compressed at a constant temperature, the pressure and volume satisfy the equation \( PV = C \), where \( C \) is a constant. Suppose that at a certain instant the volume is 400 ccs, the pressure is 80 kPa, and the pressure is increasing at a rate of 8 kPa/min.

At what rate is the volume decreasing at this instant?

1. rate = 36 ccs/min
2. rate = 40 ccs/min correct
3. rate = 42 ccs/min
4. rate = 38 ccs/min
5. rate = 34 ccs/min

**Explanation:**
After differentiation of \( PV = C \) with respect to \( t \) using the Product Rule we see that

\[ \frac{d(PV)}{dt} = P\frac{dV}{dt} + V\frac{dP}{dt} = 0, \]

which after rearrangement becomes

\[ \frac{dV}{dt} = -\frac{V}{P} \frac{dP}{dt}. \]

When

\( V = 400, \quad P = 80, \quad \frac{dP}{dt} = 8, \)

therefore,

\[ \frac{dV}{dt} = -\frac{400 \cdot 8}{80} = -40. \]

Consequently, the volume is decreasing at a rate of 40 ccs/min.

026 10.0 points

Determine \( f'(x) \) when

\[ f(x) = \sin^{-1}\left( \frac{x}{\sqrt{2 + x^2}} \right). \]

(Hint: first simplify \( f \).)

1. \( f'(x) = \frac{x}{\sqrt{2 + x^2}} \)
2. \( f'(x) = \frac{\sqrt{2}}{2 + x^2} \) correct
3. \( f'(x) = \frac{x}{x^2 + 2} \)
4. \( f'(x) = \frac{1}{2 + x^2} \)
5. \( f'(x) = \frac{\sqrt{2}}{\sqrt{2 + x^2}} \)

**Explanation:**
If \( \sin \theta = \frac{x}{\sqrt{2 + x^2}} \), then by Pythagoras’ theorem applied to the right triangle

we see that

\[ \tan \theta = \frac{x}{\sqrt{2}}. \]

Thus

\[ f(x) = \theta = \tan^{-1} \left( \frac{x}{\sqrt{2}} \right). \]

Consequently,

\[ f'(x) = \frac{\sqrt{2}}{2 + x^2}. \]

Alternatively, we can differentiate \( f \) using the Chain Rule and the fact that

\[ \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}}. \]

027 10.0 points

Find the linearization of \( f(x) = \frac{1}{\sqrt{2 + x}} \) at \( x = 0 \).

1. \( L(x) = \frac{1}{\sqrt{2}} - \frac{1}{2} x \) correct
2. \( L(x) = \frac{1}{\sqrt{2}} + \frac{1}{2} x \)
3. \( L(x) = \frac{1}{2} \left( 1 - \frac{1}{2} x \right) \)
4. \( L(x) = \frac{1}{\sqrt{2}} - \frac{1}{2} x \)
5. \( L(x) = \frac{1}{\sqrt{2}} + \frac{1}{2} x \)
6. \( L(x) = \frac{1}{2} \left( 1 - \frac{1}{2} x \right) \)

**Explanation:**

The linearization of \( f \) is the function

\[ L(x) = f(0) + f'(0)x. \]

But for the function

\[ f(x) = \frac{1}{\sqrt{2 + x}} = (2 + x)^{-1/2}, \]

the Chain Rule ensures that

\[ f'(x) = -\frac{1}{2}(2 + x)^{-3/2}. \]

Consequently,

\[ f(0) = \frac{1}{\sqrt{2}}, \quad f'(0) = -\frac{1}{4\sqrt{2}}, \]

and so

\[ L(x) = \frac{1}{\sqrt{2}} \left( 1 - \frac{1}{4} x \right). \]

028 10.0 points

Find the value of \( f'(a) \) when

\[ f(t) = \frac{2t + 1}{t + 2}. \]

1. \( f'(a) = \frac{3}{a + 2} \) correct
2. \( f'(a) = \frac{3}{(a + 2)^2} \) correct
3. \( f'(a) = -\frac{4}{a + 2} \)
4. \( f'(a) = -\frac{4}{(a + 2)^2} \)
5. \( f'(a) = -\frac{3}{(a+2)^2} \)

6. \( f'(a) = \frac{4}{(a+2)^2} \)

**Explanation:**

By definition,

\[
 f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.
\]

Now, for the given \( f \),

\[
f(a+h) = \frac{2(a+h) + 1}{a+h+2},
\]

while

\[
f(a) = \frac{2a + 1}{a+2}.
\]

Thus

\[
f(a+h) - f(a) = \frac{2(a+h) + 1}{a+h+2} - \frac{2a + 1}{a+2}
\]

\[
= \frac{(2(a+h) + 1)(a+2) - (a+h+2)(2a+1)}{(a+h+2)(a+2)}.
\]

But

\[
\{2(a+h) + 1\}(a+2) = 2h(a+2) + (2a + 1)(a+2),
\]

and

\[
(a+h+2)(2a+1) = h(2a+1) + (a+2)(2a+1).
\]

Consequently,

\[
\frac{f(a+h) - f(a)}{h} = \frac{h\{2(a+2) - (2a + 1)\}}{h(a+h+2)(a+2)}
\]

\[
= \frac{3}{(a+h+2)(a+2)},
\]

in which case

\[
f'(a) = \frac{3}{(a+2)^2}.
\]

**029 10.0 points**

Find the derivative of

\[
f(x) = x^2 \sin(x) + 2x \cos(x).
\]

1. \( f'(x) = (x^2 - 2) \cos(x) \)

2. \( f'(x) = (x^2 - 2) \sin(x) \)

3. \( f'(x) = (2 + x^2) \sin(x) \)

4. \( f'(x) = (2 - x^2) \cos(x) \)

5. \( f'(x) = (x^2 + 2) \cos(x) \) **correct**

6. \( f'(x) = (2 - x^2) \sin(x) \)

**Explanation:**

By the Product Rule

\[
\frac{d}{dx}(x^2 \sin(x)) = 2x \sin(x) + x^2 \cos(x),
\]

while

\[
\frac{d}{dx}(2x \cos(x)) = 2 \cos(x) - 2x \sin(x).
\]

Consequently,

\[
f'(x) = (x^2 + 2) \cos(x).
\]

**keywords: DerivTrig, DerivTrigExam,**

**030 (part 1 of 2) 10.0 points**

A point is moving on the graph of

\[
4x^3 + 5y^3 = xy.
\]

When the point is at

\[
P = \left( \frac{1}{9}, \frac{1}{9} \right),
\]

its \( x \)-coordinate is decreasing at a speed of 4 units per second.
What is the speed of the $y$-coordinate at that time?

1. speed $y$-coord = $-3$ units/sec
2. speed $y$-coord = $3$ units/sec
3. speed $y$-coord = $-2$ units/sec
4. speed $y$-coord = $1$ units/sec
5. speed $y$-coord = $2$ units/sec **correct**

**Explanation:**
Differentiating $4x^3 + 5y^3 = xy$ implicitly with respect to $t$ we see that

$$12x^2 \frac{dx}{dt} + 15y^2 \frac{dy}{dt} = y \frac{dx}{dt} + x \frac{dy}{dt}.$$

Thus

$$\frac{dy}{dt} = \left(\frac{12x^2 - y}{x - 15y^2}\right) \frac{dx}{dt}.$$

Now at $P\left(\frac{1}{9}, \frac{1}{9}\right)$,

$$12x^2 - y = \left(\frac{12}{(9)^2} - \frac{1}{9}\right) = \frac{1}{(9)^2}(3),$$
while

$$x - 15y^2 = \left(\frac{1}{9} - \frac{15}{(9)^2}\right) = -\frac{1}{(9)^2}(6).$$

Hence, at $P$,

$$\frac{dy}{dt} = -\frac{1}{2} \frac{dx}{dt}.$$

When the $x$-coordinate at $P$ is decreasing at a rate of 4 units per second, therefore,

$$\frac{dy}{dt} = 4\left(\frac{1}{2}\right),$$

so the

speed $y$-coord = $2$ units/sec.

---

**031 (part 2 of 2) 10.0 points**

In which direction is the $y$-coordinate moving at that time?

1. direction increasing $y$ **correct**
2. direction decreasing $y$

**Explanation:**
Since

$$\frac{dy}{dt} = 2,$$

at $P$, the $y$-coordinate of the point is moving in the

direction increasing $y$.

---

**032 10.0 points**

Find $\frac{dy}{dx}$ when

$$2x^3 + y^3 - 9xy - 1 = 0.$$

1. $\frac{dy}{dx} = \frac{2x^2 - 3y}{y^2 - 3x}$
2. $\frac{dy}{dx} = \frac{2x^2 + 3y}{y^2 - 3x}$
3. $\frac{dy}{dx} = \frac{2x^2 - 3y}{y^2 + 3x}$
4. $\frac{dy}{dx} = \frac{3y - 2x^2}{y^2 - 3x}$ **correct**
5. $\frac{dy}{dx} = \frac{3y + 2x^2}{y^2 + 3x}$

**Explanation:**
We use implicit differentiation. For then

$$6x^2 + 3y^2 \frac{dy}{dx} - 9y - 9x \frac{dy}{dx} = 0,$$
which after solving for \(\frac{dy}{dx}\) and taking out the common factor 3 gives

\[
3 \left( 2x^2 - 3y \right) + \frac{dy}{dx} (y^2 - 3x) = 0.
\]

Consequently,

\[
\frac{dy}{dx} = \frac{3y - 2x^2}{y^2 - 3x}.
\]

keywords: implicit differentiation, Folium of Descartes, derivative,

033 10.0 points

Use linear approximation to estimate the value of \(17^{1/4}\). \(\text{Hint: } (16)^{1/4} = 2.\)

1. \(17^{1/4} \approx \frac{31}{16}\)
2. \(17^{1/4} \approx \frac{63}{32}\)
3. \(17^{1/4} \approx \frac{33}{16}\)
4. \(17^{1/4} \approx 2\)
5. \(17^{1/4} \approx \frac{65}{32}\) correct

Explanation:

Set \(f(x) = x^{1/4}\), so that \(f(16) = 2\) as the hint indicates. Then

\[
\frac{df}{dx} = \frac{1}{4x^{3/4}}.
\]

By differentials, therefore, we see that

\[
f(a + \Delta x) - f(a) \approx \frac{df}{dx} \bigg|_{x=a} \Delta x = \frac{\Delta x}{4a^{3/4}}.
\]

Thus, with \(a = 16\) and \(\Delta x = 1\),

\[
17^{1/4} - 2 = \frac{1}{32}.
\]

Consequently,

\[
17^{1/4} \approx \frac{65}{32}.
\]

034 10.0 points

Find the derivative of \(g\) when

\[
g(x) = x^4 \cos(x).
\]

1. \(g'(x) = x^3 (4 \sin(x) - x \cos(x))\)
2. \(g'(x) = x^3 (4 \cos(x) - x \sin(x))\) correct
3. \(g'(x) = x^3 (4 \cos(x) + x \sin(x))\)
4. \(g'(x) = x^3 (4 \sin(x) + x \cos(x))\)
5. \(g'(x) = x^4 (3 \cos(x) - \sin(x))\)
6. \(g'(x) = x^4 (3 \sin(x) - \cos(x))\)

Explanation:

By the Product rule,

\[
g'(x) = x^4 (-\sin(x)) + (\cos(x)) \cdot 4x^3.
\]

Consequently,

\[
g'(x) = x^3 (4 \cos(x) - x \sin(x)).
\]

035 10.0 points

Find \(\frac{dy}{dx}\) when

\[
\tan(x - y) = 3x + 2y.
\]

1. \(\frac{dy}{dx} = \frac{\sec^2(x - y) + 3}{\sec^2(x - y) + 2}\)
2. \(\frac{dy}{dx} = \frac{2 - \sec^2(x - y)}{\sec^2(x - y) - 3}\)
3. \(\frac{dy}{dx} = \frac{2 + \sec^2(x - y)}{\sec^2(x - y) + 3}\)
4. \( \frac{dy}{dx} = \frac{\sec^2(x-y) - 3}{\sec^2(x-y) - 2} \)

5. \( \frac{dy}{dx} = \frac{\sec^2(x-y) - 3}{\sec^2(x-y) + 2} \) correct

6. \( \frac{dy}{dx} = \frac{2 - \sec^2(x-y)}{\sec^2(x-y) + 3} \)

Explanation:
Differentiating implicitly with respect to \( x \), we see that
\[ \sec^2(x-y)(1 - \frac{dy}{dx}) = 3 + 2\frac{dy}{dx}. \]
After rearranging, this becomes
\[ \frac{dy}{dx}(\sec^2(x-y) + 2) = \sec^2(x-y) - 3. \]
Consequently,
\[ \frac{dy}{dx} = \frac{\sec^2(x-y) - 3}{\sec^2(x-y) + 2}. \]

keywords:
036 10.0 points

If the radius of a melting snowball decreases at a rate of 1 ins/min, find the rate at which the volume is decreasing when the snowball has diameter 4 inches.

1. rate = 16\(\pi\) cu.ins/min correct
2. rate = 18\(\pi\) cu.ins/min
3. rate = 17\(\pi\) cu.ins/min
4. rate = 15\(\pi\) cu.ins/min
5. rate = 14\(\pi\) cu.ins/min

Explanation:
The volume, \( V \), of a sphere of radius \( r \) is given by
\[ V = \frac{4}{3}\pi r^3. \]
Thus by implicit differentiation,
\[ \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = -4\pi r^2, \]

since \( dr/dt = -1 \) ins/min. When the snowball has diameter 4 inches, therefore, its radius \( r = 2 \) and
\[ \frac{dV}{dt} = -4(4)\pi. \]
Consequently, when the snowball has diameter 4 inches, the volume of the snowball is decreasing at a rate = 16\(\pi\) cu.ins/min.

037 10.0 points

A point is moving on the graph of
\[ 6x^3 + 4y^3 = xy. \]
When the point is at
\[ P = \left( \frac{1}{10}, \frac{1}{10} \right), \]
its \( y \)-coordinate is increasing at a speed of 7 units per second.

What is the speed of the \( x \)-coordinate at that time and in which direction is the \( x \)-coordinate moving?

1. speed = \(\frac{9}{4}\) units/sec, increasing \( x \)
2. speed = \(\frac{17}{8}\) units/sec, decreasing \( x \)
3. speed = \(\frac{9}{4}\) units/sec, decreasing \( x \)
4. speed = 2 units/sec, increasing \( x \)
5. speed = \(\frac{7}{4}\) units/sec, increasing \( x \)
6. speed = 2 units/sec, decreasing \( x \)
7. speed = \(\frac{7}{4}\) units/sec, decreasing \( x \) correct
8. speed = \( \frac{17}{8} \) units/sec, increasing \( x \)

**Explanation:**
Differentiating

\[ 6x^3 + 4y^3 = xy \]

implicitly with respect to \( t \) we see that

\[ 18x^2 \frac{dx}{dt} + 12y^2 \frac{dy}{dt} = y \frac{dx}{dt} + x \frac{dy}{dt}. \]

Thus

\[ \frac{dx}{dt} = \left( \frac{x - 12y^2}{18x^2 - y} \right) \frac{dy}{dt}. \]

Now at \( P \),

\[ x - 12y^2 = -\frac{1}{50}, \]

while

\[ 18x^2 - y = 2 \frac{25}{25}. \]

Hence, at \( P \),

\[ \frac{dx}{dt} = -\frac{1}{4} \frac{dy}{dt}. \]

When the \( y \)-coordinate at \( P \) is increasing at a rate of 7 units per second, therefore,

\[ \frac{dx}{dt} = -\frac{7}{4}. \]

Consequently, the \( x \)-coordinate is moving at

**speed = \( \frac{7}{4} \) units/sec**, and the negative sign indicates that it is moving in the direction of decreasing \( x \).

---

2. speed = \( 15\pi \) sq. ft/sec
3. speed = 19 sq. ft/sec
4. speed = 16\( \pi \) sq. ft/sec **correct**
5. speed = 16 sq. ft/sec
6. speed = 15 sq. ft/sec
7. speed = 18 sq. ft/sec
8. speed = 17\( \pi \) sq. ft/sec

**Explanation:**
The area, \( A \), of a circle having radius \( r \) is given by \( A = \pi r^2 \). Differentiating implicitly with respect to \( t \) we thus see that

\[ \frac{dA}{dt} = 2\pi r \frac{dr}{dt}. \]

When

\[ r = 2, \quad \frac{dr}{dt} = 4, \]

therefore, the speed at which the area of the ripple is increasing is given by

**speed = 16\( \pi \) sq. ft/sec**.

---

039 10.0 points

Find \( \frac{dy}{dx} \) when

\[ \tan(xy) = x - 3y. \]

1. \( \frac{dy}{dx} = \frac{1 + y \sec^2(xy)}{x \sec^2(xy) + 3} \)
2. \( \frac{dy}{dx} = \frac{1 - y \sec^2(xy)}{x \sec^2(xy) + 3} \) **correct**
3. \( \frac{dy}{dx} = \frac{3 + x \sec^2(xy)}{y \sec^2(xy) - 1} \)
4. \( \frac{dy}{dx} = \frac{3 - x \sec^2(xy)}{y \sec^2(xy) - 1} \)

---

038 10.0 points

A rock is thrown into a still pond and causes a circular ripple. If the radius of the ripple is increasing at a rate of 4 ft/sec, at what speed is the area of the ripple increasing when its radius is 2 feet?

1. speed = 18\( \pi \) sq. ft/sec
5. \( \frac{dy}{dx} = \frac{1 - y \sec^2(xy)}{x \sec^2(xy) - 3} \)

6. \( \frac{dy}{dx} = \frac{3 - x \sec^2(xy)}{y \sec^2(xy) + 1} \)

**Explanation:**
Differentiating implicitly with respect to \( x \), we see that
\[
\sec^2(xy) \left( y + x \frac{dy}{dx} \right) = 1 - 3 \frac{dy}{dx}.
\]
After rearranging, this becomes
\[
\frac{dy}{dx} \left( x \sec^2(xy) + 3 \right) = 1 - y \sec^2(xy).
\]
Consequently,
\[
\frac{dy}{dx} = \frac{1 - y \sec^2(xy)}{x \sec^2(xy) + 3}.
\]

**keywords:**

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Find the derivative of
\[
g(x) = \left( \frac{x + 2}{x + 3} \right) (2x - 3).
\]

1. \( g'(x) = \frac{2x^2 - 12x - 9}{x + 3} \)

2. \( g'(x) = \frac{2x^2 + 12x + 9}{x + 3} \)

3. \( g'(x) = \frac{x^2 - 12x + 9}{x + 3} \)

4. \( g'(x) = \frac{x^2 + 12x - 9}{(x + 3)^2} \)

5. \( g'(x) = \frac{2x^2 + 12x + 9}{(x + 3)^2} \) correct

6. \( g'(x) = \frac{2x^2 - 12x - 9}{(x + 3)^2} \)

**Explanation:**

By the Quotient and Product Rules we see that
\[
g'(x) = 2 \left( \frac{x + 2}{x + 3} \right) + (2x - 3) \left( \frac{(x + 3) - (x + 2)}{(x + 3)^2} \right)
\]
\[
= 2 \left( \frac{x + 2}{x + 3} \right) + \left( \frac{2x - 3}{(x + 3)^2} \right)
\]
\[
= \frac{2(x + 2)(x + 3) + (2x - 3)}{(x + 3)^2}.
\]
But
\[
2(x + 2)(x + 3) + (2x - 3)
\]
\[
= 2x^2 + 12x + 9.
\]
Consequently
\[
g'(x) = \frac{2x^2 + 12x + 9}{(x + 3)^2}.
\]

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Find \( dy/dx \) when
\[
3x^2 + 2y^2 = 5.
\]

1. \( \frac{dy}{dx} = \frac{x}{2y} \)

2. \( \frac{dy}{dx} = \frac{3x}{2y} \)

3. \( \frac{dy}{dx} = -\frac{3x}{2y} \) correct

4. \( \frac{dy}{dx} = -\frac{3x}{y} \)

5. \( \frac{dy}{dx} = -3xy \)

6. \( \frac{dy}{dx} = 2xy \)

**Explanation:**

Differentiating
\[
3x^2 + 2y^2 = 5
\]
implicitly with respect to \( x \) we see that

\[
6x + 4y \frac{dy}{dx} = 0.
\]

Consequently,

\[
\frac{dy}{dx} = -\frac{6x}{4y} = -\frac{3x}{2y}.
\]

042 10.0 points

A balloon is released 3 feet away from an observer. The balloon is rising vertically at a rate of 3 ft/sec and at the same time the wind is carrying it horizontally away from the observer at a rate of 2 ft/sec. At what speed is the angle of inclination of the observer's line of sight changing 3 seconds after the balloon is released?

1. speed = \( \frac{2}{27} \) rads/sec
2. speed = \( \frac{1}{27} \) rads/sec
3. speed = \( \frac{5}{54} \) rads/sec
4. speed = \( \frac{1}{18} \) rads/sec correct
5. speed = \( \frac{1}{9} \) rads/sec

Explanation:
Let \( y = y(t) \) be the height of the balloon \( t \) seconds after release and let \( x = x(t) \) be its horizontal distance from the point of release as shown in the figure.

Then by right triangle trigonometry, the angle of inclination \( \theta \) satisfies the equation

\[
\tan(\theta) = \frac{y}{x + 3}.
\]

Differentiating this equation implicitly with respect to \( t \) we see that

\[
\sec^2(\theta) \frac{d\theta}{dt} = \frac{(x + 3)\frac{dy}{dt} - y\frac{dx}{dt}}{(x + 3)^2}.
\]

But \( \sec^2(\theta) = 1 + \tan^2(\theta) \). Thus

\[
\frac{d\theta}{dt} = \frac{(x + 3)\frac{dy}{dt} - y\frac{dx}{dt}}{(x + 3)^2 + y^2} = \frac{3(x + 3) - 2y}{(x + 3)^2 + y^2} \text{ rads/sec}.
\]

Now after 3 seconds,

\[
x(3) = 3 \frac{dx}{dt} = 6 \text{ ft/sec},
\]

while

\[
y(3) = 3 \frac{dy}{dt} = 9 \text{ ft/sec}.
\]

Hence after 3 seconds the speed at which the angle of inclination is changing is given by

\[
\text{speed} = \frac{1}{18} \text{ rads/sec}.
\]

043 10.0 points

The dimensions of a cylinder are changing, but the height is always equal to the diameter of the base of the cylinder. If the height is increasing at a speed of 5 inches per second, determine the speed at which the volume, \( V \), is increasing (in cubic inches per second) when the height is 2 inches.

1. \( \frac{dV}{dt} = 14 \pi \text{ cub. ins./sec} \)
2. \[ \frac{dV}{dt} = 16\pi \text{ cub. ins./sec} \]

3. \[ \frac{dV}{dt} = 12\pi \text{ cub. ins./sec} \]

4. \[ \frac{dV}{dt} = 15\pi \text{ cub. ins./sec correct} \]

5. \[ \frac{dV}{dt} = 13\pi \text{ cub. ins./sec} \]

Explanation:
Since the height \( h \) of the cylinder is equal to its diameter \( D \), the radius of the cylinder is \( r = \frac{1}{2}h \). Thus, as a function of \( h \), the volume of the cylinder is given by

\[ V(h) = \pi r^2 h = \frac{\pi}{4} h^3. \]

The rate of change of the volume, therefore, is

\[ \frac{dV}{dt} = \frac{3\pi}{4} \left( h^2 \frac{dh}{dt} \right). \]

Now

\[ \frac{dh}{dt} = 5 \text{ ins./sec.} \cdot \]

Consequently, when \( h = 2 \text{ ins.} \),

\[ \frac{dV}{dt} = 15\pi \text{ cub. ins./sec}. \]

Find \( dy/dx \) when \( y + x = 6\sqrt{xy} \).

1. \[ \frac{dy}{dx} = \frac{\sqrt{\frac{y}{x}} - 3}{6 + \sqrt{\frac{x}{y}}} \]

2. \[ \frac{dy}{dx} = \frac{3\sqrt{\frac{y}{x}} + 1}{1 - 3\sqrt{\frac{x}{y}}} \]

3. \[ \frac{dy}{dx} = \frac{3\sqrt{\frac{y}{x}} - 1}{1 - 3\sqrt{\frac{x}{y}}} \text{ correct} \]

4. \[ \frac{dy}{dx} = \frac{3\sqrt{\frac{y}{x}} - 1}{1 + 3\sqrt{\frac{x}{y}}} \]

5. \[ \frac{dy}{dx} = \frac{\sqrt{\frac{y}{x}} + 3}{6 - \sqrt{\frac{x}{y}}} \]

6. \[ \frac{dy}{dx} = \frac{\sqrt{\frac{y}{x}} + 3}{6 + \sqrt{\frac{x}{y}}} \]

Explanation:
Differentiating implicitly with respect to \( x \), we see that

\[ \frac{dy}{dx} + 1 = 3 \left( \sqrt{\frac{y}{x}} + \sqrt{\frac{x}{y}} \frac{dy}{dx} \right), \]

so

\[ \frac{dy}{dx} \left( 1 - 3\sqrt{\frac{x}{y}} \right) = 3\sqrt{\frac{y}{x}} - 1. \]

Consequently,

\[ \frac{dy}{dx} = \frac{3\sqrt{\frac{y}{x}} - 1}{1 - 3\sqrt{\frac{x}{y}}}. \]

Find an equation for the tangent line to the graph of \( f(x) = \sec(x) + 2\cos(x) \) at the point \( \left( \frac{1}{3}\pi, f\left( \frac{1}{3}\pi \right) \right) \).

1. \( 2y - 3\sqrt{3}x = 1 - \sqrt{3}\pi \)

2. \( 2y + 3\sqrt{3}x = \sqrt{3}\pi - 1 \)

3. \( 2y - \sqrt{3}x = 3 - \frac{1}{\sqrt{3}}\pi \)
4. \( y - \sqrt{3}x = 3 - \frac{1}{\sqrt{3}}\pi \) correct

5. \( y - 3\sqrt{3}x = 1 - \sqrt{3}\pi \)

6. \( y + 3\sqrt{3}x = \sqrt{3}\pi - 1 \)

Explanation:
Since \( f\left(\frac{1}{3}\pi\right) = 3 \), we have to find an equation for the tangent line to the graph of \( f \) at the point \( P\left(\frac{1}{3}\pi, 3\right) \). Now
\[
f'(x) = \sec(x)\tan(x) - 2\sin(x),
\]
so the slope at \( P \) is
\[
f'(\frac{1}{3}\pi) = \sqrt{3}.
\]
By the point-slope formula, therefore, an equation for the tangent line at \( P \) is given by
\[
y - 3 = \sqrt{3}\left(x - \frac{1}{3}\pi\right),
\]
which after rearrangement and simplification becomes
\[
y - \sqrt{3}x = 3 - \frac{1}{\sqrt{3}}\pi.
\]

046 10.0 points

Find the derivative of
\[
f(x) = \frac{1}{3}\left(\arctan(3x)\right)^2.
\]

1. \( f'(x) = \frac{1}{1 + 9x^2} \arctan(3x) \)
2. \( f'(x) = \frac{2}{1 + 9x^2} \arctan(3x) \) correct
3. \( f'(x) = \frac{1}{9 + x^2} \arctan(3x) \)
4. \( f'(x) = \frac{2}{9 + x^2} \arctan(3x) \)
5. \( f'(x) = \frac{1}{3} \sec^2(3x) \tan(3x) \)

6. \( f'(x) = 2\sec^2(3x) \tan(3x) \)

Explanation:
Since
\[
\frac{d}{dx} \arctan(x) = \frac{1}{1 + x^2},
\]
the Chain Rule gives
\[
\frac{d}{dx} \arctan(3x) = \frac{3}{1 + 9x^2}.
\]
Using the Chain Rule yet again, therefore, we see that
\[
f'(x) = \frac{2}{1 + 9x^2} \arctan(3x).
\]

keywords: derivative, inverse tan, Chain Rule,

047 10.0 points

A street light is on top of a 12 foot pole. A person who is 5 feet tall walks away from the pole at a rate of 3 feet per second. At what speed is the length of the person’s shadow growing?

1. speed = 2 ft/sec
2. speed = \( \frac{15}{7} \) ft/sec correct
3. speed = \( \frac{16}{7} \) ft/sec
4. speed = \( \frac{17}{7} \) ft/sec
5. speed = \( \frac{18}{7} \) ft/sec

Explanation:
If \( x \) denotes the length of the person’s shadow and \( y \) denotes the distance of the person from the pole, then shadow and the lightpole are related in the following diagram
By similar triangles,

\[
\frac{5}{x} = \frac{12}{x+y},
\]

so \(5y = (12 - 5)x\). Thus, after implicit differentiation with respect to \(t\),

\[
5\frac{dy}{dt} = (12 - 5)\frac{dx}{dt}.
\]

When \(dy/dt = 3\), therefore, the length of the person’s shadow is growing with

\[
\text{speed} = \frac{15}{7} \text{ ft/sec}.
\]