Dynamical Systems
Graduate Course M 393C, 56105, 17347, Fall 2014

Instructor

François Baccelli: baccelli@math.utexas.edu, RLM 11.136.

Objectives

The course will be of introductory nature. The requirements are limited to basic knowledge in topology, measure theory, functional analysis, and probability. The course will be focused on discrete time dynamical systems and will cover both the deterministic and the stochastic settings. The main basic notions of the theory will be presented and several classical examples will be discussed.

The instructional objective is that students be in a position to start mathematical research on dynamical systems and to use the described methodology in applications.

Contents

I. Deterministic Dynamical Systems

The basic notions to be covered are:

- Topological dynamics: ω and α-limit sets, topological transitivity, recurrence, attractors, topological entropy, topological conjugacy, factors.
- Ergodic theory: measure preserving transformations, Von Neumann's mean ergodic theorem, Birkhoff's pointwise ergodic theorem, Poincaré's recurrence theorem, mixing, measure theoretic entropy.

Several examples will be discussed including linear maps, expanding maps, translations of the torus, symbolic dynamics, and hyperbolic automorphisms on the torus. This part of the course will be based on material from the following books:

[KH] A. Katok & B. Hasselblatt, *Introduction to the Modern Theory of Dynamical Systems* Cambridge University Press, 1995.

[Pe] K. Petersen, *Ergodic theory*, Cambridge University Press, 1989.

[BT] H. Broer & F. Takens, *Dynamical Systems and Chaos*, Springer, 2010.

II. Stochastic Dynamical Systems

The main basic notions to be covered are:

- Stochastic recursive systems and pathwise representation of Markov chains;
- Construction of factors by coupling from the past: Loynes' construction, and the Propp and Wilson construction.

Several examples will be discussed: Markov chains, products of random matrices, iterates of non-expansive random maps, and stochastic dynamics in queuing theory. For this part of the course, the following material will be used:

[BB] F. Baccelli & P. Brémaud, *Elements of Queueing Theory*, Springer Verlag, second edition, 2003.

[Bo] A. Borovkov, *Ergodicity and Stability of Stochastic Processes*, Wiley, 1998.

Grading Policy

- Assignments: 1/3;
- 2 midterm exams (tentatively October 14 and November 25 at the time and place of the class): 1/3;
- One research paper to read and present (from a list of proposed papers) at the end of the semester: 1/3.

Practical Information

Class Hours: TTH 3.30-5.00pm RLM 12.166.

Office Hours: Tentatively, the instructor F. Baccelli will have his office hours on TTH 2.00-3.30 pm in RLM 11.136.

Course Policy: Course material will be available on Blackboard.

Accommodations: Students with disabilities may request appropriate academic accommodations from the Division of Diversity and Community Engagement, Services for Students with Disabilities, 512-471-6259, http://www.utexas.edu/diversity/ddce/ssd/

Core Objectives: This course may be used to fulfill the mathematics component of the university core curriculum and addresses the following three core objectives established by the Texas Higher Education Coordinating Board: communication skills, critical thinking skills, and empirical and quantitative skills.