A. Galois - Nearby Hecke Factors & K-Theory Methods

6/24/00

X = k(x1) \rightarrow \text{agt}(X) \times \text{agt}(X) \text{ central extension, study mod-1}

so that \text{agt}(X) \times \text{agt}(X) \text{ acts continuously (module by discrete topology). Study similarities with p-adic groups...}

Bernstein center: according to local Langlands philosophy, has to do with functions on \text{Gal}(k(x1) \rightarrow \mathbb{C}.

\text{A mod-1 system on space X.)}

Bernstein version: moduli L5 of \text{agt}(X) \text{ of local systems for k on Spec} X.

Bernstein center should relate to functions on L5.

Consider representations with level integral and critical. Both Bernstein center of this category & functions on L5 are trivial!!

H-C for f.d. \text{agt}(X[x]) \times \mathbb{Z}[x]

Verve modules depending on parameter give functions on here, intertwines give invariance, this gives too invariant:

for \text{agt}(X) \text{ of center gives function on deck to center & get \text{Weil} functions, noncritical \text{ lattice out by translations} \rightarrow \text{no such functions}.

Consider bundles with regular singularity: up to isomorphisms come from \text{agt}(X) \text{ consider residue of the covering element of some space as before. But some transformations give action of lattice again no invariants, so triviality for same reasons in both cases.}

\rightarrow \text{try to reformulate}.

• Category has spectral decomposition wrt center - can localize over spec (center) "spectral theorem for category".

\text{agt}(k) = \text{mod} has a "spectral decomposition" wrt \text{agt}(k).

What does \text{agt}(k) \text{ look like? Can consider families of local systems - structure of quotient of intuitive ind-scheme of id finite type by an id proper equivalence relation."

"id": union of schemes and family of closed embeddings.

Example: like bundles \text{L} \text{ has C. Consider trivialized line bundle...}
described by the "potential" $\in \text{Spec}(\mathcal{O}(X))$, modulo gauge equivalence by $K^*$.  
- depends only on polar part: 
  $\mathcal{O}(X)/K^* = (\mathcal{O}(X)/K^*)/(X^*/O^*)$  
  (affine vector space)  

$X^*/O^*$: $\mathbb{Z}$ merely ... but already parameters get  
  $\mathbb{Z} \times (X/O)$  
  formal group of $X/O$.  

$\mathcal{O}(X)/\mathcal{O}(O)$: $k \times d(X/O)$  
  (formal power)  

$\mathbb{Z}$ acts by translating on $k$, $(X/O)^*$ acts by translation  
on $d(X/O)$.  
- vector space / its formal group...  

Quasi-coherent sheaves on this: representations of Heisenberg  
system $(X/O \times \mathbb{C})^* \rightarrow \mathcal{O}(x/O)$  
$\mathbb{Z} \subset \mathcal{O}$ translating, $X/O$ give vector fields.  

- formal commutative geometry.  

Don't know similar description for general $C$ - only know its  
points! irregular part comes from some after finite cover ...  
object collection of components looking like  
commutative case, but intersect in complicated ways!  

Rough definition & spectral description: For $R$ comm. alg.,  
$Y: \text{Spec } R \rightarrow \mathbb{S}_{\mathbb{C}}$, $R$-point:  

assign a category of $R$-modules with  
$\mathcal{O}(X)^*$-action, factorial w/ $R$.  
(should evolve to  
map $R \rightarrow$ Berns..-center in usual way...).  
Ind-setting: should twist $R$-modules to $\text{Diff}(R^n)$  
+ structure: replace pullback(??) by $C$!...  

$p$-adic setting: have Tate isom for unramified reps.  

Global picture: for any rep of $G$ to  
to extend to global setting: try to extend to aut-every $K$  
rep of full group $\rightarrow$ global adele $K$'s rep... can look  
best at local components.  

Kac-Moody setting: can consider living on "larger" disc  
then $p$-adic setting -- don't need global automorphic  
setting...
Have Satake in Deligne setting
V is structure; can reproduce Hecke action.

Satake picture: spherical Hecke algebra \( \leftrightarrow \text{Rep } \mathcal{G} \).
\( \mathcal{G}_\mathcal{E} \) was perverse sheaves on \( \mathcal{G}_\mathcal{E} \), \( \text{tr}(\text{Fr}_\mathcal{E} \otimes \mathbb{R}) \rightarrow \text{im} \).

Pass to D-modules, twisted by \( \text{loc} \) bundle \( \leftrightarrow \) level
so global sections are \( \text{og}(\mathcal{X})^\mathbb{R} \)-modules.
\( \Rightarrow \) Functor \( \text{Rep } \mathcal{G} \rightarrow \text{og}(\mathcal{X})^\mathbb{R} \)-mod

On isomorphism classes: multiply h.w. of \( V \in \text{Rep } \mathcal{G} \)
by level, get weight for \( \mathcal{G} \), take level h.w. of \( \text{og}(\mathcal{X})^\mathbb{R} \)
-- get \( \text{Rep } \) small reps this way.

(\text{level} \Leftrightarrow \text{invariant quadratic form, } x \rightarrow x^2, \text{cohomology w.s.})
These reps \( \leftrightarrow \text{Dmod on } \mathcal{G}(\mathbb{K})/\mathcal{G}(\mathbb{O}) \) inside \( \mathcal{G}(\mathbb{K})/\mathcal{G}(\mathbb{O}) \) D-mod.
get extra structure \( \mathcal{G}(\mathbb{K}) \), integrable, \( \mathcal{G}(\mathbb{O}) \), non-rectifiable.
Negative level! V as \( \infty \)-finite, length,
positive: \( "\text{finite length} \)"

Hecke functor \( V \in \text{Rep } \mathcal{G} \), \( \mathcal{H}(V) \) corresponding \( \text{og}(\mathcal{X})^\mathbb{R} \)-module
\( x \in \mathcal{X} \) corresponds to \( x \in (x, \text{og}(\mathcal{X})^\mathbb{R}) \)-mod

\( x \) varies: get D-module on \( \mathcal{X} \).
\( x \) parameter = \( x \Rightarrow \) Lie algebra \( \text{og}[x, t][t^{-1}] \) localize \( x \rightarrow t^{-1} \).
get action compatible with containing one above.

\( M \in \text{C}(\text{og}(\mathcal{X})^\mathbb{R}) \)-mod, \( 0 \leq \mathcal{X} \), \( x = x_0 \). \( U = x \cdot 0 \rightarrow \mathcal{X} \)
\( M \otimes j^* \mathcal{H}(V) \) (cohom at 0) \( \mathcal{D}-\text{mod} \)
has action of \( \text{og}[x, t][t^{-1}] \) on \( \mathcal{H}(V) \)
\( \text{og}[x, t][t^{-1}]((t)) \Rightarrow \otimes \text{og}[x^\mathbb{R} \otimes \mathbb{C}(\mathcal{X})]
\)
\( \Rightarrow \text{og}[x^\mathbb{R} \otimes \mathbb{C}(\mathcal{X})] \)
\( \mathcal{E}(V) \)

Consider max ind \( \mathcal{D}-\text{mod} \) quotient
of \( M \otimes j^* \mathcal{H}(V) \) supported at \( x \)

\( \text{b invariant wrt this Lie algebra action} \)
\( \Rightarrow \) action of \( \text{og}[x, t][t^{-1}] \) \( \text{mod } x = \text{og}[t, t^{-1}] \)
which is continuous \[
\text{get functor } M \to \mathfrak{L}(V)M = \text{this vector space with } \text{og}(x) \text{-action}
\]
\[V = \text{trivial rep } \Rightarrow \text{identity functor}
\]

Con tensor by local systems: \[\mathcal{L}(V) \text{ defined in some way but start with } M \to \mathfrak{L}(V)M \text{ -- control all possible variations.}
\]

Rough property of compatibility of spectral decomposition with the \[\mathfrak{L}(V)\]:

\[M \text{ comes from category of reps coming to } \mathcal{G} \text{ R-Rep}(\mathcal{L})\]

\[\text{Want } M \to \mathfrak{L}(V)M \text{ of } \mathcal{V}_p
\]

System of such with compatibilities

For each loc sys consider fiber of category! class of modules over it...

If loc sys has regular regularities:

should be D-mod on additive flags with all possible

Coh in twists...

Given a rep, take its support on \[\mathcal{L}\text{ that reduce to tori over } K \text{ not an exception:}

\text{restrict mod to } \mathcal{O}(K), \text{ take cohomology}

\text{by module at Hecke ring of tori, order sum over } F \text{ flags -- this should be support.}

Remark: this picture for purely unipotent local system.