$U(k,d) = \text{semitable rank } k \text{ degree } d \text{ bundle on } C$

Rank 1 case: $\text{Jac}^d(C)\rightarrow \Theta = \{ L \in k^* : h^0(L) > 0 \}$

$\text{Jac}^d(C)$ has no canonical $k$-module, but if we fix a line bundle $M$ on $C$
with $\mathcal{K}(L \otimes M) = 0$ left of degree $d$

$\Rightarrow \Theta_{k,M} = \{ L \in k^* : h^0(L \otimes M) > 0 \} < \mathcal{J}^d$

Hilbert $\text{rank}: E \in U(k,k-1) \Rightarrow \mathcal{K}(E) = 0$

$\Rightarrow U(k,k-1) \supset \Theta_k = \{ E : h^0(E) > 0 \}$

For $U(k,d) > \Theta_k,E = \{ F : h^0(E \otimes F) > 0 \}$

where $E$ is chosen so that $\mathcal{K}(E \otimes F) = 0$

$\forall F \in U(k,d)$

get a divisor for generic $E$

With $E$ having slope $(g-1) - \frac{d}{k}$
So minimal rank for $E$ is $\mu \geq c(d,E)$.

Now for $\text{det}$: $\text{SU}(k,n) \rightarrow \text{U}(k)$.;

$\text{Pic}(\text{SU}) = \mathbb{Z}$. For $E$ of minimal rank

$\Theta_k, c \mid_{\text{SU}}$ is an ample generator for $\text{Pic}(\text{SU})$.

Strange duality, degree zero case:

Let $\tau: \text{SU}(r,1) \times \text{U}(k,k-g) \rightarrow \text{U}(r,k, r(k-1))$

$E \mapsto E \otimes \Theta_{r-k}$

$\tau^* \Theta_{r-k} = \Theta_{r-k} \otimes \Theta_{r-k}$ as line bundles

On space of sections:

$H^0(\text{SU}(r,1), \text{U}(k,k-g), \tau^* \Theta_{r-k})$

$= H^0(\text{SU}(r,1), \Theta_{r-k}) \otimes H^0(\text{U}(k,k-g), \Theta_{r-k})$

but have enough sections $\tau^*(\Theta_{r-k})$. 
... more to come \{ (E,F) \ s.t. \ h^0(E \otimes F) \neq 0 \}

\Rightarrow SD: \ h^0(\mathbb{C}U(1), \theta_i)_\mathbb{R} \to \mathbb{R} \cup \{0, \infty\}, \theta_i^b)

end at \ E \to \mathbb{R}_+, \ E

Conjecture: SD is an isomorphism

(Steinberg) ... Volterra formula in dimension m+1...

Proof for generic case by Pechersk

for all cases by Vanishing Green

... want to find enough pairs \ E_i, \theta_i \ (1 \leq i \leq 9)

s.t. \ \theta_k, \ E_i (F_j) \neq 0 \iff i = j.

ie. \ h^0(\mathbb{C}U(1), \theta \otimes F_j) = 0 \iff i = j.

... trying to prove \theta_i \ s.p.n

so need a good interpretation of the Volterra number

\ h^0(\mathbb{C}U(1), \theta_i^b) = K(\mathbb{C}U(1), \theta_i^b)

\ h^0(\mathbb{C}U(1), \theta_i^b) = \frac{\delta_i}{(2\pi)^d} \int_{\mathbb{R}^{2d}} \chi_{\mathbb{R}^{2d}}(t) \chi_t(\mathbb{R}^{2d}) (W(k))
Intersection number on the Quot scheme with an enumerative meaning...

Without loss of generality, find

$$\int_{Quot_{h,k}} \omega = h^o(U(k,0), \Theta_{k,m} \otimes dd^c \Theta_{1,0})$$

where $M,L$ live bundles of deg 5-1

$$= h^o(U(r,0), \Theta_{r,0} \otimes dd^c \Theta_{1,0})$$

So let's reformulate the duality in terms of

the bundles involved, more here nicer

$$\Delta_{L,M} = \{ (E,F) s.t. h^o(E \oplus F \oplus M) \neq 0$$

$$\wedge$$

or $h^o(\text{det} E \otimes \text{det} F \otimes L \otimes K) = 0 \}$$

$$U(r)^{\times} U(k,0)$$
\[ \text{in } \sigma : U(r, 0) \times U(k, 0) \longrightarrow U(k, 0) \times V \]

\[ E \circ F \longrightarrow E \circ F \text{ def. } E \circ \text{def} F \]

\[ \text{so } \quad U(\Delta_{2, m}) = \sigma^* ( \theta_{c, m} \otimes \theta_i, \text{def} \theta_{c, m} ) \]

\[ D : \quad \text{H}^0 (U(n, 0), \theta_r, m \otimes \text{def} \theta_{c, m}, \nu) \longrightarrow \]

\[ \text{H}^0 (U(n, 0), \theta_r, m \otimes \text{def} \theta_{c, m}, l \otimes \omega) \]

**Theorem**

\[ \text{D is an isomorphism} \]

\[ \text{idea: see: map } E, F \quad \text{set } \Delta_{2, m} (k) \rightarrow \text{H}^0 \]

**Theorem**

\[ \text{stage 1, 2, 3: must check conditions to} \]

\[ \text{find, determine locus is surjective at } E \circ F \text{.} \]

\[ \text{Q.w.d. (U, r, k, C) proper since } k \text{.} \]

\[ \text{irreducible for } d \text{; } \quad r \rightarrow F \rightarrow 0 \rightarrow k \text{.} \]

\[ \text{map } k \rightarrow \text{def } r \rightarrow 0 \]

\[ d \rightarrow k \text{.} \]
Universal objects: \( \mathcal{E} \to \mathcal{O}^{\mathbb{A}^1 - 1} \)

\[
\begin{array}{l}
\text{Class } a_k = C_{k_0} \left( E_k^{(0)} \right) \quad p \in \mathbb{C} \\
\text{assume degree } d \geq 0 (k)
\end{array}
\]

What is the power \( a_k^{\text{deg}} = a_k^s \)?

\( F \ni P_i, \ldots P_s \in C \) & generic lies in \((\mathcal{O}^{\mathbb{A}^1})^s = V \)

\( \Rightarrow \) we're considering \( \left( a_k \right) \text{ exact sequence} \)

\[
\begin{array}{c}
0 \to E_i \to D_k^m \to F_i \to 0
\end{array}
\]

\( S.t. \quad L_j \otimes D_i \to \mathcal{O}^{\mathbb{A}^1 - 1} \to E_i \)

is zero at \( p_0 \)

Equivalently we're considering exact sequence

\[
\begin{array}{c}
0 \to E_i \to S \to F_i' \to 0
\end{array}
\]

\( d_1, d_3, d_5, d_7 \)
where $S$ is such $\eta$ \[ \begin{array}{c} \eta \colon \mathfrak{g} \rightarrow \mathfrak{g} \end{array} \]

But this is what we want! $h^0(\mathcal{F}_1 \otimes \mathcal{F}_2) \neq 0$

If it is $\ldots$ then $E \otimes \det F_i = \det S$

$E \otimes (\mathcal{F}_1 \otimes \mathcal{F}_2)$ is sure it's the

Zariski tangent of our Ordinal

intersection in the next place $\blacksquare$

interested in ancient interpretation of Volterra matrix

Witt's paper Volume Algebra & Quantum Cohomology of the Grassmann

Zagier: $h^0(S(V, v), \Theta^2) =$

255 $\sum_{v \in E} \left( \begin{array}{c} 255 \alpha_1 + \frac{5}{\alpha_1 + k} \end{array} \right)$
Veek - Intrillignt

\[ \sum_{\text{over } k-tuples} \prod_{(p,q)} \text{if } p+q \neq 1 \]

... comes from localization

- looks like RRS of Zarou, some