Dehn Surgery and 3-Manifolds Exercise Set #5

Exercise 1: What 3-manifold is this?

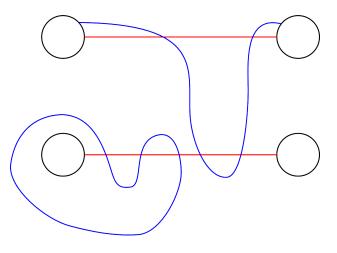


Figure 1: Heegaard diagram example

<u>Exercise 2:</u> Let m_1 , l_1 , m_2 , and l_2 be integers such that $m_1l_2 - m_2l_1 = 1$. Show that $S^3_{m_1/l_1}(K)$, $S^3_{m_2/l_2}(K)$, and $S^3_{m_3/l_3}(K)$ form a triad where $m_3 = m_1 + m_2$ and $l_3 = l_1 + l_2$.

Exercise 3: Suppose (M_A, M_B, M_C) is a triad such that

$$|H_1(M_A)| + |H_1(M_B)| = |H_1(M_C)|.$$

Show that if M_A and M_B are L-spaces then so is M_C . Hint: Use rank-nullity.

Exercise 4: Let n > 0 be an integer such that for some knot, K, $S_n^3(K)$ is an L-space. Show that for every rational slope $r \ge n$, $S_r^3(K)$ is an L-space.

<u>Exercise 5:</u> Let K_1 and K_2 be non-trivial knots in S^3 . Show that no surgery on $K_1 \# K_2$ can yield the Poincaré homology sphere.