Heegaard Floer Homology Exercise Set #1

Exercise 1: Show that

$$\frac{H_1(\mathsf{Sym}^g(\Sigma))}{H_1(\mathbb{T}_\alpha) \oplus H_1(\mathbb{T}_\beta)} \cong \frac{H_1(\Sigma)}{[\alpha_1] \cdots [\alpha_g][\beta_1] \cdots [\beta_g]} \cong H_1(Y)$$

Exercise 2: Argue that for a genus 1 pointed Heegaard diagram $(\Sigma, \alpha, \beta, z)$ of a lens space or S^3 (i. e. not $S^1 \times S^2$), $\widehat{HF}$ is invariant under isotopy or changing of the base point.

Exercise 3: Compute $\widehat{HF}(L(5,2))$. Generalize to any lens space $L(p,q)$. ($L(5,2)$ is pictured below.)
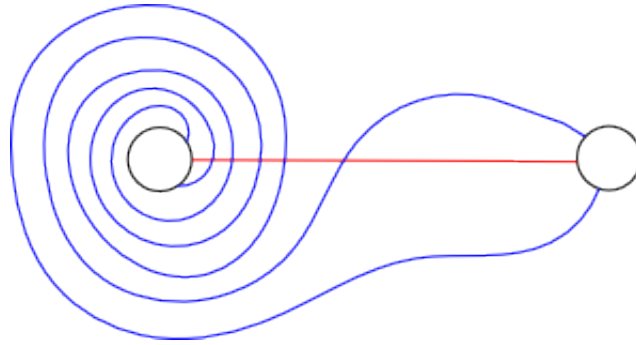

Figure 1: Heegaard diagram of $L(5,2)$

Exercise 4: Find all of the points in $\mathbb{T}_\alpha \cap \mathbb{T}_\beta$ in the following Heegaard diagram. **Bonus:** What manifold is this?
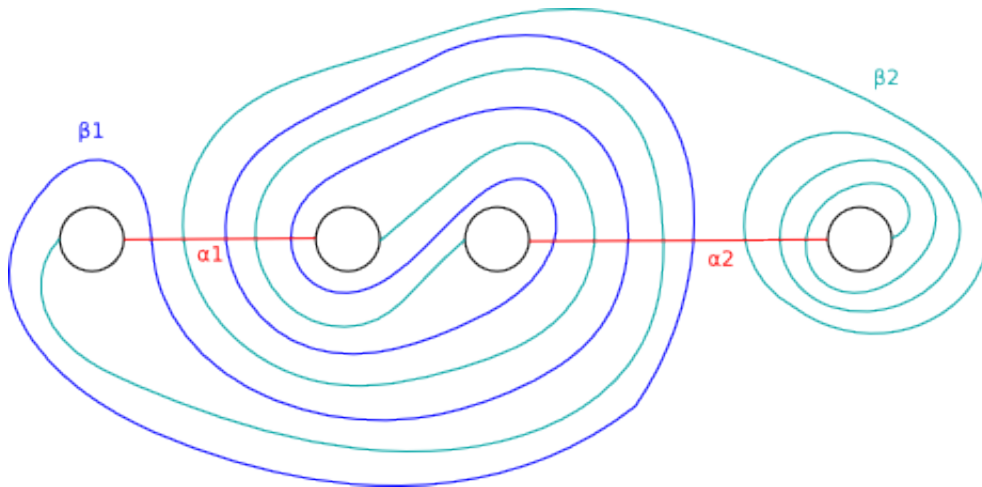

Figure 2: Heegaard diagram of a familiar 3-manifold.