

Exercise Set #1

Exercise 1: Use Poincaré-Lefschetz duality to prove that following about Euler characteristic χ :

- (a) $\chi(M) = 0$ for any closed orientable 3-manifold M .
- (b) $2\chi(M) = \chi(\partial M)$ for any compact orientable 3-manifold M with boundary ∂M .

Exercise 2: Show that the fundamental group of a 3-manifold with Heegaard genus g has a group presentation with g generators and g relators.

Exercise 3: Let Σ_g be a closed orientable surface of genus g . Find a Heegaard splitting for $\Sigma_g \times S^1$.

Exercise 4: Suppose the 3-manifolds M_1 and M_2 have Heegaard genera g_1 and g_2 . Show that the Heegaard genus of $M_1 \# M_2$ is $g_1 + g_2$.

Exercise 5: Find a Heegaard splitting of \mathbf{RP}^3 .

Exercise 6: Prove that any two Heegaard splittings for a 3-manifold M given by triangulations are stably equivalent.