EXERCISE SET # 4

<u>Exercise 1:</u> For $\mathcal{L}$ a framed link in S^3, prove that the linking matrix of $\mathcal{L}$ is a presentation matrix for $H^2(S^3(\mathcal{L}); \mathbb{Z}) \cong H_1(S^3(\mathcal{L}); \mathbb{Z})$.

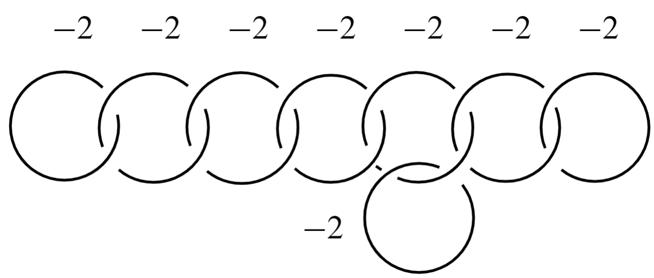
<u>Exercise 2:</u> For R a commutative ring with identity, call a 3-manifold Y an $R-$homology S^3 (RHS^3) if $H_*(Y; R) \cong H_*(S^3; R)$.

- Prove that any $(\mathbb{Z}/2\mathbb{Z})HS^3$ is orientable.
- Let Y be a $\mathbb{Q}HS^3$, and let A and B be simply-connected 4-manifolds with $\partial A = -\partial B = Y$, and form the closed 4-manifold $X = A \cup_Y B$. Show that $Q_A \oplus Q_B \hookrightarrow Q_X$, and that if A and B are both negative definite then so is X.

<u>Exercise 3:</u> Prove that neither of the following lens spaces bound a $\mathbb{Q}HB^4$.

- $L(25, 18)$
- $L(2, 1)$

<u>Exercise 4:</u> Certify that the trace of surgery on the following framed link yields a negative definite 4-manifold with boundary the Poincaré homology sphere $\mathcal{P}$. Prove that $\mathcal{P}$, with *this* orientation, does not bound a positive-definite 4-manifold.



<u>Exercise 5:</u> Construct a cobordism $X : \mathcal{P} \to L(5, 4)\#L(3, 2)\#L(2, 1)$.

<u>Exercise 6:</u> Prove that the total space of the disc bundle associated to the tangent bundle of S^2 is diffeomorphic to the trace of $+2$-surgery on the unknot in S^3 (up to orientation).