Perpetual American Options; Introduction to Exotic Options

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Definition; Optimal Exercise

- Because of the possibility of early exercise, it is sensible to define American options with infinite maturity
- They are called perpetual options or expirationless options
- Because there are no obstacles produced by the finite horizon (maturity time), the valuation formula is available for these options
- In order to price the options, we need to start by establishing the conditions for optimal early exercise:

  Perpetual American options are optimally exercised when the underlying asset reaches the optimal exercise barrier H^*

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Barrier Present Values

- Let H be **any** (barrier) level
- We call the barrier present value the value today of the \$1 at time when the stock price process S hits the level H
- There are simple formulas for the barrier present value:

  If $H > S$, then it equals $(S/H)^{h_1}$ with

$$h_1 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}.$$

  If $H < S$, then it equals $(S/H)^{h_2}$ with

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Perpetual Calls

- The optimal exercise barrier is H^* given by

$$H^* = K\left(\frac{h_1}{h_1 - 1}\right)$$

- If $\delta = 0$, then $H^* = \infty$
- The price is

$$\frac{K}{h_1 - 1}\left(\frac{h_1 - 1}{h_1} \cdot \frac{S}{K}\right)^{h_1}$$

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Perpetual American Options; Introduction to Exotic Options

Definition

- Another name for exotic options that are not of the American/European type that we mainly discussed so far is **nonstandard** options
- Exotic options solve particular business problems that an ordinary option cannot
- They are constructed by tweaking ordinary options in minor ways
- Some relevant questions:

1. How does the exotic payoff compare to ordinary option payoff?
2. Can the exotic option be approximated by a portfolio of other options?
3. Is the exotic option cheap or expensive relative to standard options? Does this question make sense? What are we comparing here?
4. What is the rationale for the use of the exotic option?
5. How easily can the exotic option be hedged?

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Asian Options

- The payoff of an Asian option is path dependent
- More precisely, it is based on the **average** price over some period of time
- There are various ways in which one can interpret the word "average" and that needs to be postulated in the option contract
- Some examples of situations when Asian options are useful are:

1. When a business cares about the average exchange rate over time
2. When a single price at a point in time might be subject to manipulation
3. When price swings are frequent due to thin markets (low liquidity markets)

- A practical example would be the exercise of the conversion option in convertible bonds as it is based on the stock price over a 20-day period at the end of the bonds life
- Asian options are less valuable than otherwise identical ordinary options

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Basic Kinds of Asian Options

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- There are eight basic kinds of Asian options:

1. Put or call
2. Geometric or arithmetic average
3. Average asset price is used in place of underlying price or strike

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Types of Averages

Arithmetic versus geometric average

Suppose we record the stock price at periods of length h between times 0 and T

- Arithmetic average:

$$A_T = \frac{1}{N} \sum_{i=1}^{N} S_{ih}$$

- Geometric average:

$$G_T = \left(\prod_{i=1}^{N} S_{ih} \right)^{\frac{1}{N}}$$

- It is always true that $G_T \leq A_T$

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Usage of Averages

Average as asset price versus average as strike price

- Average used as the asset price $\longrightarrow$ **Average price option**
- Call payoff:

$$(A_T - K)^+ \text{ or } (G_T - K)^+$$

- Put payoff:

$$(K - A_T)^+ \text{ or } (K - G_T)^+$$

- Average used as the strike price $\longrightarrow$ **Average strike option**
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- You can read about it in Appendix 14.A

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An Example

Let us observe the following set-up throughout one year:

- An international firm **XYZ** has monthly revenue of 100 million euros, and costs in US dollars
- Let x_i be the spot dollar price of a euro in month i
- At the end of the one year, the converted amount in dollars is

$$100 \times 10^6 \times \sum_{i=1}^{12} x_i e^{1-\frac{i}{12}}$$

- Let us ignore the interest in the expression above; then, the amount that needs to be hedged (due to the changes in the interest rates) is

$$\sum_{i=1}^{12} x_i = 12 \left(\frac{\sum_{i=1}^{12} x_i}{12} \right)$$

- A convenient option for the above task would be an Asian put with a certain floor K (on the average rate received); its payoff is

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$$\left(K - \frac{\sum_{i=1}^{12} x_i}{12} \right)^+$$

An Example

Let us observe the following set-up throughout one year:

- An international firm **XYZ** has monthly revenue of 100 million euros, and costs in US dollars
- Let x_i be the spot dollar price of a euro in month i
- At the end of the one year, the converted amount in dollars is

$$100 \times 10^6 \times \sum_{i=1}^{12} x_i e^{1-\frac{i}{12}}$$

- Let us ignore the interest in the expression above; then, the amount that needs to be hedged (due to the changes in the interest rates) is

$$\sum_{i=1}^{12} x_i = 12 \left(\frac{\sum_{i=1}^{12} x_i}{12} \right)$$

- A convenient option for the above task would be an Asian put with a certain floor K (on the average rate received); its payoff is

$$\left(K - \frac{\sum_{i=1}^{12} x_i}{12} \right)^+$$