UNIVERSITY OF TEXAS AT AUSTIN

Project # 3

Power of a $z-$test. $t-$test.

Problem 3.1. (20 points) **Power of a $z-$test.**

Example 6.15 (page 383 from the textbook) gives a test of a hypothesis about the mean consumption of sugar-sweetened beverages at your university based on a sample of size of 100. In this case well assume that the population distribution is normal and that the population standard deviation is given at 155 calories. The hypotheses are

$$H_0 : \mu = 286 \quad vs. \quad H_a : \mu \neq 286$$

The sample average was 262, so while the result was not statistically significant at the significance levels smaller than the obtained $p-$value of 0.12, it did provide some evidence that the population mean was *smaller* than 286.

Thus, you plan to recruit another sample of students from your university but **this time use a one-sided alternative**. You were thinking of surveying 100 students, but now wonder if this sample size gives adequate power.

(i) (12 points) What is the *rejection region* under the above null hypothesis for a one-sided alternative? Complete the following steps in R:
- (3 points) Draw the density of the **sampling distribution** of the sample mean under the null hypothesis.
- (2 points) Draw the vertical line indicating the **mean** of the population distribution under the null hypothesis (preferably in a different color).
- (2 points) Draw the vertical line indicating the **upper bound** of the rejection region for a significance level of 0.05 (preferably in a different color).
- (5 points) Shade the region below the normal density function to the left of the upper bound you found in the previous task.

(ii) (8 points) What is the correspondence between the alternative values of the population mean and the power of the above test?
- (5 points) Define a **function** which will calculate (from first principles) the power of the above test as a function of the alternative population mean.
- (3 points) Draw the graph of the function you obtained in the previous task.

Problem 3.2. (24 points) **$t-$test.**

In the **Odin.R** code, we started by creating an entire population by drawing the simulations from an exponential distribution. Then, we proceeded to draw simple random samples from the population.

An equivalent approach, would be to draw the simple random samples by simulating them directly from a particular distribution. So, let us now assume that the population distribution is normal, centered at zero and with the standard deviation equal to 2.

(i) (2 points) Draw one SRS of size 100 from the above distribution and display the histogram of the SRS.

(ii) (4 points) Find the $p-$value of your sample for the two-sided test with the null hypothesis that the population mean equals zero.

(iii) (14 points) Set the significance level at 0.05. Test the above hypothesis for 1000 SRSs and find the proportion of **rejections** in those 1000 tests. Report your findings.

(iv) (4 points) Repeat the previous task for sigificance levels 0.01 and 0.10.