UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 4

Bernoulli. Binomial.

Provide your **complete solutions** for the following problems.

Problem 4.1. Based on the traveling salesman's experience, he makes a same on any visit with probability of 15%. We assume that the individual customer's decisions are independent.

If he makes 10 visits in a certain day, what is the chance that he makes at least five sales?

Solution: Let Y denote the number of sales he makes. Then $Y \sim Binomial(n = 10, p = 0.15)$.

Note

$$\mathbb{P}[Y \geq 5] = 1 - \mathbb{P}[Y \leq 4].$$

So,

$$\begin{aligned}\mathbb{P}[Y \leq 4] &= \mathbb{P}[Y = 0] + \cdots + \mathbb{P}[Y = 4]\\ &= \binom{10}{0}(0.15)^0(0.85)^{10} + \binom{10}{1}(0.15)^1(0.85)^9 + \binom{10}{2}(0.15)^2(0.85)^8 + \binom{10}{3}(0.15)^3(0.85)^7 + \binom{10}{4}(0.15)^4(0.85)^6\\ &= \cdots = 0.99012.\end{aligned}$$

The answer is $\mathbb{P}[Y \geq 5] = 0.0099$.

Problem 4.2. Expected frequency

Suppose you are going to roll a fair die 60 times and record the proportion of times that a 1 or a 2 is showing. The sampling distribution of the said proportion should be centered about which value?

Solution: 1/3

We model every roll in the repeated $n-$tuple of trials as a single Bernoulli trial. So, in this particular problem, we have $X_i, i = 1, \ldots, n$ to be independent, identically distributed with

$$X_i \sim Bernoulli(p = 1/3).$$

Note that $\mathbb{E}[X_i] = p = 1/3$ for all i.

The statistics denoting the proportion of "successes" in the repeated trials is defined as

$$\hat{p} = \frac{X_1 + X_2 + \cdots + X_n}{n}.$$

The center of the *sampling distribution* of $\hat{p}$ is exactly its expected value. So, in the present problem,

$$\mathbb{E}[\hat{p}] = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i] = \frac{n\mathbb{E}[X_1]}{n} = \mathbb{E}[X_1] = p = 1/3.$$