

## UNIVERSITY OF TEXAS AT AUSTIN

## Problem Set # 9

The  $t$ -procedure: two means9.1. Two-sample  $t$ .

**Problem 9.1.** Let the population distributions be normal with unknown parameters. Assume that sample data, based on two independent samples of size 25, give us  $\bar{x}_1 = 505$ ,  $\bar{x}_2 = 515$ ,  $s_1 = 23$ , and  $s_2 = 28$ .

- (i) What is a 95%-confidence interval (use the conservative value for the degrees of freedom) for the difference between the two population means?
- (ii) Based on the confidence interval, we can conclude at the 5% significance level that there is no difference between the two population means. *True or false?*
- (iii) The margin of error for the difference between the two sample means would be smaller if we were to take larger samples. *True or false?*
- (iv) If a 99% confidence interval were calculated instead of the 95% interval, it would include more values for the difference between the two population means. *True or false?*

**Solution:**

- (i) We get

$$s^2 = \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} = \frac{23^2 + 28^2}{25} = \frac{1313}{25} \Rightarrow s = 7.247.$$

The critical  $t^*$  corresponds to the upper-tail probability of 0.025 and the number of degrees of freedom equal to 24. We get  $t^* = 2.064$ . So, the 95%-confidence interval is

$$505 - 515 \pm 2.064(7.247) = -10 \pm 14.9578$$

- (ii) **TRUE**
- (iii) **TRUE**
- (iv) **TRUE**

9.2. Pooled  $t$ .

**Problem 9.2.** The **pooled** two-sample  $t$ -procedure can be used when ...

- (a) you can assume the two populations have equal variances
- (b) you can assume the two populations have equal means
- (c) the sample sizes are equal
- (a) None of the above

**Solution: (a)**

**Problem 9.3.** Let  $n_1$  and  $n_2$  denote the sample sizes of each group. The pooled two-sample  $t$ -procedure is based how many degrees of freedom?

- (a)  $n_1 + n_2 + 2$
- (b)  $n_1 + n_2 - 2$
- (c)  $n_1 + n_2 - 1$
- (d)  $n_1 + n_2$
- (e) None of the above.

**Solution: (b)**

**Problem 9.4.** A study was done to determine if students learn better in an online basic statistics class versus a traditional face-to-face (f2f) course. A random sample of 12 students in an online course and 15 students in an f2f course was taken.

- (i) Let  $\mu_{new}$  denote the population mean score for the online statistics class and let  $\mu_{old}$  denote the population mean score for the face-to-face statistics class. What are the hypotheses being tested?

**Solution:**

$$H_0 : \mu_{new} = \mu_{old} \quad vs. \quad H_a : \mu_{new} > \mu_{old}$$

- (ii) We decide it is appropriate to use the pooled  $t$ -procedure. What is the number of degrees of freedom you are going to use?

**Solution:**

$$12 + 15 - 2 = 25.$$

**Problem 9.5.** This is an excerpt from findings of an educational study:

A study was done to determine whether there is a difference in the amount of time (in hours) that graduate students versus undergraduate students spend on the Internet per day. Five undergrads and five grad students were polled.

- (i) Is the alternative hypothesis one-sided or two-sided?

**Solution:** It's two-sided.

- (ii) A  $t$ -score for the data gathered was calculated to be 1.6664. Would you say that there is a significant difference in the amount of time that graduate and undergraduate students spend on the Internet?

**Solution:** No.

**Problem 9.6.** An instructor is teaching two sections of the same basic statistics course. The instructor is giving the same exams, homework assignments, and quizzes in both sections. Which  $t$ -procedure should be used to determine if there is a difference in the academic performance between the two course sections?

- (a) One-sample  $t$ -test.
- (b) Matched-pairs  $t$ -procedure.
- (c) Two-sample  $t$ -test.
- (d) None of the above.

**Solution:** (c)